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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M2
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Inquiry and Investigation

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. Let $S_n = \sum_{k=1}^{n} k\,2^k$ for a positive integer n. By evaluating S_1, S_2, S_3, conjecture a closed form for S_n in terms of n, and verify your conjecture for $n = 4$. **(3 marks)**

2. A student claims: "If a and b are irrational numbers, then $a + b$ is irrational." Either prove the claim or disprove it by a counter-example, and justify your conclusion clearly. **(3 marks)**

3. A sequence is defined by $u_1 = 1$ and $u_{n+1} = \dfrac{u_n}{2u_n + 1}$ for $n \geq 1$. Find u_2, u_3, u_4, conjecture a formula for u_n, and prove it by induction. **(4 marks)**

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4. Let $f(x) = x^3 + px + q$ where p, q are real constants, and suppose f has a local maximum value M and a local minimum value m. **(6 marks)**

(a) Show that $M - m = \frac{4}{3}(-\frac{p}{3})^{3/2}$ when $p < 0$.

(b) Hence determine, with justification, the condition on p and q for the equation $f(x) = 0$ to have exactly three distinct real roots.

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SECTION A(2) (24 marks)

5. A square of side 2 has a smaller square of side x cut from each corner, and the remaining cross-shaped net is folded up to form an open box of height x. **(7 marks)**

(a) Write down the volume $V(x)$ of the box and state the domain of x.

(b) Find the value of x that maximises V, and the maximum volume.

(c) A student claims the maximum volume occurs when the base is a square of side 1. Examine this claim and comment.

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6. **(8 marks)**

Define a sequence by $a_1 = 2$ and $a_{n+1} = \dfrac{a_n^2 + 2}{2a_n}$ for $n \geq 1$.

(a) Prove by induction that $a_n > \sqrt{2}$ for all $n \geq 1$.

(b) Prove that $a_{n+1} < a_n$ for all $n \geq 1$.

(c) Explain why (a_n) converges, and find its limit.

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7. Let $I_n = \int_0^{\pi/2} \sin^n x \, dx$ for integers $n \geq 0$. **(9 marks)**

(a) Using integration by parts, show that $I_n = \frac{n-1}{n} I_{n-2}$ for $n \geq 2$.

(b) Hence evaluate I_5 and I_6.

(c) Show that $I_{2n+1} < I_{2n} < I_{2n-1}$ and deduce that $\lim_{n \to \infty} \frac{I_{2n+1}}{I_{2n}} = 1$.

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SECTION B (10 marks)

8. For a positive integer n, let $P(n)$ be the statement: "$7^n + 3^n$ is divisible by 10." **(10 marks)**

(a) Show that $P(n)$ is true for $n = 1$ and $n = 2$.

(b) A student attempts a proof by induction: "Assume $7^k + 3^k$ is divisible by 10. Then $7^{k+1} + 3^{k+1} = 7 \cdot 7^k + 3 \cdot 3^k$. Since $7^k + 3^k \equiv 0 \pmod{10}$, we get $7^{k+1} + 3^{k+1} \equiv 7 \cdot 7^k + 3 \cdot 3^k \equiv \dots$" Complete the argument correctly, or show that the induction step as started fails, and give a valid proof.

(c) Determine, with justification, all positive integers n for which $7^n + 3^n$ is divisible by 10.

END OF PAPER

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