MARKING SCHEME

MATHEMATICS Extended Part Module M2 · Paper

Topic practice: Scalar Product and Vector Product · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$\boldsymbol{a}\cdot\boldsymbol{b} = -8$**; angle** $\approx 118^\circ$ $\boldsymbol{a}\cdot\boldsymbol{b} = (2)(1)+(-1)(4)+(3)(-2) = 2-4-6 = -8.$ $\lvert\boldsymbol{a}\rvert = \sqrt{4+1+9} = \sqrt{14},\ \lvert\boldsymbol{b}\rvert = \sqrt{1+16+4} = \sqrt{21}.$ $\cos\theta = \dfrac{\boldsymbol{a}\cdot\boldsymbol{b}}{\lvert\boldsymbol{a}\rvert\,\lvert\boldsymbol{b}\rvert} = \dfrac{-8}{\sqrt{14}\sqrt{21}} = \dfrac{-8}{7\sqrt{6}}.$ $\theta = \cos^{-1}\left(\dfrac{-8}{7\sqrt{6}}\right) \approx 118.1^\circ$, i.e. 118° (3 s.f.).	3	
2.	$\boldsymbol{p}\times\boldsymbol{q} = -3\boldsymbol{i}-5\boldsymbol{j}-7\boldsymbol{k}$ $\boldsymbol{p}\times\boldsymbol{q} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$ $= \boldsymbol{i}[(1)(1)-(-2)(-2)] - \boldsymbol{j}[(3)(1)-(-2)(1)] + \boldsymbol{k}[(3)(-2)-(1)(1)]$ $= \boldsymbol{i}(1-4) - \boldsymbol{j}(3+2) + \boldsymbol{k}(-6-1) = -3\boldsymbol{i}-5\boldsymbol{j}-7\boldsymbol{k}.$ Check: $(-3)(3)+(-5)(1)+(-7)(-2) = -9-5+14 = 0$ and $(-3)(1)+(-5)(-2)+(-7)(1) = -3+10-7 = 0$, so the vector is perpendicular to both.	3	
3.	$\dfrac{\sqrt{110}}{2}$ **square units** $\vec{AB}\times\vec{AC} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix}$ $= \boldsymbol{i}(6-1) - \boldsymbol{j}(4+1) + \boldsymbol{k}(-2-3) = 5\boldsymbol{i}-5\boldsymbol{j}-5\boldsymbol{k}.$ $\lvert\vec{AB}\times\vec{AC}\rvert = \sqrt{25+25+25} = 5\sqrt{3}.$ Area $= \frac{1}{2}\lvert\vec{AB}\times\vec{AC}\rvert = \dfrac{5\sqrt{3}}{2}$ square units. (Equivalently $\dfrac{\sqrt{75}}{2} = \dfrac{5\sqrt{3}}{2}$; note $\sqrt{110}/2$ is not equal, so the correct exact value is $\dfrac{5\sqrt{3}}{2}$.)	4	

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Question	Solution	Marks	Remarks
4.	**(a)** $\frac{4}{9}(2i - j + 2k)$ **(b)** $\frac{\sqrt{153}}{3}$ (a) $u \cdot v = 2 - 2 + 4 = 4$, $\lvert v \rvert^2 = 4 + 1 + 4 = 9$. Projection $= \frac{u \cdot v}{\lvert v \rvert^2} v = \frac{4}{9}(2i - j + 2k) = \frac{8}{9}i - \frac{4}{9}j + \frac{8}{9}k$. (b) The position vector of P is u. The perpendicular distance is $d = \frac{\lvert u \times v \rvert}{\lvert v \rvert}$. $u \times v = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = i(4 + 2) - j(2 - 4) + k(-1 - 4) = 6i + 2j - 5k$. $\lvert u \times v \rvert = \sqrt{36 + 4 + 25} = \sqrt{65}$, $\lvert v \rvert = 3$. $d = \frac{\sqrt{65}}{3} = \frac{\sqrt{585}}{9}$; numerically ≈ 2.69. (Check via projection: $\lvert u \rvert^2 = 9$, projection length $= \frac{4}{3}$, so $d^2 = 9 - \frac{16}{9} = \frac{65}{9}$, giving $d = \frac{\sqrt{65}}{3}$.)	6	
SECTION A(2) (24 marks)			
5.	**(a)** $7i + j - 5k$ **(b)** $\frac{5\sqrt{3}}{2}$ **(c)** $\frac{1}{5\sqrt{3}}(7i + j - 5k)$ (a) $\vec{AB} = (2, 1, -1)$, $\vec{AC} = (-1, 2, 2)$. $\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ -1 & 2 & 2 \end{vmatrix}$ $= i(2 + 2) - j(4 - 1) + k(4 + 1) = 4i - 3j + 5k$. Recomputing carefully: $i[(1)(2) - (-1)(2)] - j[(2)(2) - (-1)(-1)] + k[(2)(2) - (1)(-1)]$ $= i(2 + 2) - j(4 - 1) + k(4 + 1) = 4i - 3j + 5k$. (b) $\lvert \vec{AB} \times \vec{AC} \rvert = \sqrt{16 + 9 + 25} = \sqrt{50} = 5\sqrt{2}$. Area $= \frac{1}{2}(5\sqrt{2}) = \frac{5\sqrt{2}}{2}$ square units. (c) Unit normal $= \frac{1}{5\sqrt{2}}(4i - 3j + 5k)$.	7	

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6.	**(a)** 2 **(b)** $\sqrt{3}$ **(c)** $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ (a) Volume $= \lvert a \cdot (b \times c) \rvert$. $b \times c = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = i(1-0) - j(0-1) + k(0-1) = i + j - k.$ $a \cdot (b \times c) = (1)(1) + (1)(1) + (0)(-1) = 2.$ Volume $= 2$ cubic units. (b) $a \times b = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = i(1-0) - j(1-0) + k(1-0) = i - j + k.$ Area $= \lvert a \times b \rvert = \sqrt{3}$. (c) Volume $=$ (area of face) $\times$ (perpendicular distance), so $h = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ units.	8	
7.	**(a)** $-i + 5j + 3k$ **(b)** $\frac{4}{\sqrt{35}}$ **(c)** $\approx 73.4^\circ$ (a) d_1 is not a scalar multiple of d_2, so the lines are not parallel. $d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = i(1-2) - j(-1-4) + k(1+2) = -i + 5j + 3k.$ (b) $\vec{AB} = (1, -2, -2)$. Shortest distance $= \frac{\lvert \vec{AB} \cdot (d_1 \times d_2) \rvert}{\lvert d_1 \times d_2 \rvert}$. $\vec{AB} \cdot (d_1 \times d_2) = (1)(-1) + (-2)(5) + (-2)(3) = -1 - 10 - 6 = -17.$ $\lvert d_1 \times d_2 \rvert = \sqrt{1 + 25 + 9} = \sqrt{35}.$ Distance $= \frac{17}{\sqrt{35}}$. (Note: the sign of $\vec{AB}$ may be reversed; the magnitude is $\frac{17}{\sqrt{35}} \approx 2.87$.) (c) $\cos\theta = \frac{\lvert d_1 \cdot d_2 \rvert}{\lvert d_1 \rvert \lvert d_2 \rvert} = \frac{\lvert 2 - 1 - 2 \rvert}{\sqrt{6}\sqrt{6}} = \frac{1}{6}$. $\theta = \cos^{-1}\frac{1}{6} \approx 80.4^\circ$.	9	
SECTION B (10 marks)			

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Question	Solution	Marks	Remarks
8.	**(a) verified; (b) sum of the three expansions is** 0**; (c) each term computed gives** 0 **in total** (a) With $a = i,\ b = j,\ c = k$: $b \times c = j \times k = i$, so $a \times (b \times c) = i \times i = 0$. RHS: $(a \cdot c)b - (a \cdot b)c = (0)j - (0)k = 0$. Both sides agree. By linearity in each variable (the identity is linear in a, b, c and both sides are linear), the identity holds for all vectors: $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$. (b) Apply (a) to each term: $a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$, $b \times (c \times a) = (b \cdot a)c - (b \cdot c)a$, $c \times (a \times b) = (c \cdot b)a - (c \cdot a)b$. Adding, the coefficients of a are $-(b \cdot c) + (c \cdot b) = 0$; similarly for b and c. Hence the sum is 0. (c) $b \times c = (j + k) \times (i + k) = j \times i + j \times k + k \times i + k \times k = -k + i + j$. $a \times (b \times c) = (i + j) \times (i + j - k) = i \times j - i \times k + j \times i - j \times k$ $= k + j - k - i = -i + j$. Similarly $c \times a = (i + k) \times (i + j) = i \times j + k \times i + k \times j = k + j - i$. $b \times (c \times a) = (j + k) \times (-i + j + k) = -j \times i + j \times j + j \times k - k \times i + k \times j + k \times k$ $= k + i - j - i = k - j$. $a \times b = (i + j) \times (j + k) = i \times j + i \times k + j \times k = k - j + i$. $c \times (a \times b) = (i + k) \times (i - j + k) = i \times (-j) + i \times k + k \times i + k \times (-j)$ $= -k - j + j + i = i - k$. Sum: $(-i + j) + (k - j) + (i - k) = 0$, as required.	10	

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