MARKING SCHEME

MATHEMATICS Extended Part Module M2 · Paper

Topic practice: Introduction to Vectors · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$9i - 19j$ $2a = 2(3i - 2j) = 6i - 4j$. $3b = 3(-i + 5j) = -3i + 15j$. $2a - 3b = (6i - 4j) - (-3i + 15j) = (6 + 3)i + (-4 - 15)j = 9i - 19j$.	3	
2.	$\begin{pmatrix} 12 \\ -11 \end{pmatrix}$ $3p = 3\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$. $2q = 2\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$. $3p - 2q = \begin{pmatrix} 6 \\ -3 \end{pmatrix} - \begin{pmatrix} -6 \\ 8 \end{pmatrix} = \begin{pmatrix} 6 + 6 \\ -3 - 8 \end{pmatrix} = \begin{pmatrix} 12 \\ -11 \end{pmatrix}$.	3	
3.	$\frac{7}{2}i + j$ The position vector of the midpoint M is $\vec{OM} = \frac{1}{2}(\vec{OA} + \vec{OB})$. $\vec{OM} = \frac{1}{2}[(2i + 3j) + (5i - j)] = \frac{1}{2}(7i + 2j) = \frac{7}{2}i + j$.	4	
4.	**(a)** $\lvert u \rvert = \sqrt{5}$, $\lvert v \rvert = \sqrt{10}$. **(b)** $\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (a) $\lvert u \rvert = \sqrt{1^2 + 2^2} = \sqrt{5}$. $\lvert v \rvert = \sqrt{3^2 + (-1)^2} = \sqrt{10}$. (b) $u + v = \begin{pmatrix} 1 + 3 \\ 2 - 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. $\lvert u + v \rvert = \sqrt{4^2 + 1^2} = \sqrt{17}$. The unit vector is $\frac{1}{\sqrt{17}}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.	6	
SECTION A(2) (24 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
5.	**(a)** $\vec{AD} = -a + \frac{1}{4}b$, $\vec{BC} = -\frac{2}{3}a + b$. **(b)** $\lambda = \frac{8}{11}$, $\mu = \frac{3}{11}$. **(c)** $AP:PD = 8:3$. (a) $\vec{OC} = \frac{2}{3}a$, $\vec{OD} = \frac{1}{4}b$. $\vec{AD} = \vec{OD} - \vec{OA} = \frac{1}{4}b - a = -a + \frac{1}{4}b$. $\vec{BC} = \vec{OC} - \vec{OB} = \frac{2}{3}a - b = -\frac{2}{3}a + b$. (b) $\vec{OP} = \vec{OA} + \vec{AP} = a + \lambda(-a + \frac{1}{4}b) = (1-\lambda)a + \frac{\lambda}{4}b$. Also $\vec{OP} = \vec{OB} + \vec{BP} = b + \mu(-\frac{2}{3}a + b) = -\frac{2\mu}{3}a + (1+\mu)b$. Equating coefficients: $1 - \lambda = -\frac{2\mu}{3}$ and $\frac{\lambda}{4} = 1 + \mu$. From the second equation, $\lambda = 4 + 4\mu$. Substituting into the first: $1 - (4 + 4\mu) = -\frac{2\mu}{3} \Rightarrow -3 - 4\mu = -\frac{2\mu}{3} \Rightarrow -9 - 12\mu = -2\mu \Rightarrow -9 = 10\mu \Rightarrow \mu = -\frac{9}{10}$. Then $\lambda = 4 + 4(-\frac{9}{10}) = 4 - \frac{36}{10} = \frac{4}{10} = \frac{2}{5}$. Wait, check: $\lambda = 4 + 4\mu = 4 - 3.6 = 0.4 = \frac{2}{5}$. But then $\vec{OP} = (1 - \frac{2}{5})a + \frac{1}{10}b = \frac{3}{5}a + \frac{1}{10}b$. Also from the second expression: $-\frac{2(-0.9)}{3}a + (1 - 0.9)b = 0.6a + 0.1b$, consistent. So $\lambda = \frac{2}{5}$, $\mu = -\frac{9}{10}$. But μ should be positive if P lies on segment BC? Actually P is intersection of lines, may lie outside segments. The problem states lines AD and BC intersect at P, so μ can be negative. (c) $AP:PD = \lambda:(1-\lambda) = \frac{2}{5}:\frac{3}{5} = 2:3$.	7	
6.	**(a)** $\vec{OD} = 4i + 5j$. **(b)** $ABCD$ **is a rhombus because** $\lvert\vec{AB}\rvert = \lvert\vec{BC}\rvert = 3\sqrt{2}$. **(c)** $\vec{OE} = 3i$. (a) For parallelogram $ABCD$, $\vec{AB} = \vec{DC}$. $\vec{AB} = \vec{OB} - \vec{OA} = (4i - j) - (i + 2j) = 3i - 3j$. $\vec{DC} = \vec{OC} - \vec{OD} \Rightarrow \vec{OD} = \vec{OC} - \vec{AB} = (7i + 2j) - (3i - 3j) = 4i + 5j$. (b) $\lvert\vec{AB}\rvert = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$. $\vec{BC} = \vec{OC} - \vec{OB} = (7i + 2j) - (4i - j) = 3i + 3j$. $\lvert\vec{BC}\rvert = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$. Since $ABCD$ is a parallelogram with $\lvert\vec{AB}\rvert = \lvert\vec{BC}\rvert$, it is a rhombus. (c) $\vec{OE} = \vec{OA} + \frac{2}{3}\vec{AB} = (i + 2j) + \frac{2}{3}(3i - 3j) = i + 2j + 2i - 2j = 3i$.	8	
7.	**(a)** 3. **(b)** $\sqrt{19}$. **(c)** 101.5°. (a) $a \cdot b = \lvert a\rvert\,\lvert b\rvert\cos 60^\circ = (2)(3)(\frac{1}{2}) = 3$. (b) $\lvert a+b\rvert^2 = \lvert a\rvert^2 + 2a\cdot b + \lvert b\rvert^2 = 4 + 6 + 9 = 19$, so $\lvert a+b\rvert = \sqrt{19}$. (c) $\lvert a-b\rvert^2 = \lvert a\rvert^2 - 2a\cdot b + \lvert b\rvert^2 = 4 - 6 + 9 = 7$, so $\lvert a-b\rvert = \sqrt{7}$. $(a+b)\cdot(a-b) = \lvert a\rvert^2 - \lvert b\rvert^2 = 4 - 9 = -5$. Let θ be the angle. Then $\cos\theta = \frac{-5}{\sqrt{19}\sqrt{7}} = \frac{-5}{\sqrt{133}}$. $\theta = \cos^{-1}(\frac{-5}{\sqrt{133}}) \approx 101.5^\circ$.	9	
SECTION B (10 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
8.	**(a)** $\vec{AG} = -\frac{2}{3}a + \frac{1}{3}b$, $\vec{OG} = \frac{1}{3}(a+b)$. **(b)** P **is midpoint of** AB. **(c)** $OG:GP = 2:1$. (a) Let $\vec{AG} = \lambda\vec{AN}$ and $\vec{BG} = \mu\vec{BM}$. $\vec{AN} = \vec{ON} - \vec{OA} = \frac{1}{2}b - a$. $\vec{BM} = \vec{OM} - \vec{OB} = \frac{1}{2}a - b$. $\vec{OG} = \vec{OA} + \vec{AG} = a + \lambda(\frac{1}{2}b - a) = (1-\lambda)a + \frac{\lambda}{2}b$. Also $\vec{OG} = \vec{OB} + \vec{BG} = b + \mu(\frac{1}{2}a - b) = \frac{\mu}{2}a + (1-\mu)b$. Equating coefficients: $1-\lambda = \frac{\mu}{2}$ and $\frac{\lambda}{2} = 1-\mu$. From second, $\lambda = 2 - 2\mu$. Substitute into first: $1-(2-2\mu) = \frac{\mu}{2} \Rightarrow -1+2\mu = \frac{\mu}{2} \Rightarrow -1 = -\frac{3\mu}{2} \Rightarrow \mu = \frac{2}{3}$. Then $\lambda = 2 - \frac{4}{3} = \frac{2}{3}$. So $\vec{AG} = \frac{2}{3}(\frac{1}{2}b - a) = -\frac{2}{3}a + \frac{1}{3}b$. $\vec{OG} = (1-\frac{2}{3})a + \frac{1}{3}b = \frac{1}{3}a + \frac{1}{3}b = \frac{1}{3}(a+b)$. (b) P lies on AB, so $\vec{OP} = (1-t)a + tb$ for some t. Also P lies on line OG, so $\vec{OP} = k\vec{OG} = \frac{k}{3}(a+b)$. Equating: $1-t = \frac{k}{3}$ and $t = \frac{k}{3}$. Thus $1-t = t \Rightarrow t = \frac{1}{2}$. So $\vec{OP} = \frac{1}{2}(a+b)$, which is the midpoint of AB. (c) $\vec{OG} = \frac{1}{3}(a+b)$ and $\vec{OP} = \frac{1}{2}(a+b)$. $\vec{GP} = \vec{OP} - \vec{OG} = (\frac{1}{2} - \frac{1}{3})(a+b) = \frac{1}{6}(a+b)$. Thus $OG:GP = \frac{1}{3}:\frac{1}{6} = 2:1$.	10	

TUTGATHER