MARKING SCHEME

MATHEMATICS Extended Part Module M2 · Paper

Topic practice: Matrices · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$\begin{pmatrix} 1 & -2 \\ 12 & -7 \end{pmatrix}$ $2A = \begin{pmatrix} 4 & -2 \\ 6 & 8 \end{pmatrix}$ and $3B = \begin{pmatrix} 3 & 0 \\ -6 & 15 \end{pmatrix}$. Thus $2A - 3B = \begin{pmatrix} 4-3 & -2-0 \\ 6-(-6) & 8-15 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 12 & -7 \end{pmatrix}$.	3	
2.	$\frac{1}{14}\begin{pmatrix} 5 & -3 \\ -2 & 4 \end{pmatrix}$ The determinant of C is $\det C = (4)(5) - (3)(2) = 20 - 6 = 14$. Since $\det C \neq 0$, C is invertible and $C^{-1} = \frac{1}{14}\begin{pmatrix} 5 & -3 \\ -2 & 4 \end{pmatrix}$.	3	
3.	**Both sides equal** $\begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$**.** First compute $PQ = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1+6 & 0+2 \\ 0+3 & 0+1 \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$, so $(PQ)^T = \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$. Next, $Q^T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ and $P^T = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. Then $Q^T P^T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1+6 & 0+3 \\ 0+2 & 0+1 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$. Hence $(PQ)^T = Q^T P^T$.	4	
4.	**(a)** $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} = -2I$**; (b)** $B = \frac{1}{2}(5I - A) = \begin{pmatrix} 2 & -1 \\ -1.5 & 0.5 \end{pmatrix}$**.** (a) $A^2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$. Also $5A = \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix}$. Thus $A^2 - 5A = \begin{pmatrix} 7-5 & 10-10 \\ 15-15 & 22-20 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$. Wait, the answer given is $-2I$; let me recompute: $A^2 - 5A = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I$. So the correct result is $2I$, not $-2I$. I will correct the answer and solution accordingly. (b) From (a), $A^2 - 5A = 2I$, so $A(A - 5I) = 2I$, i.e. $A \cdot \frac{1}{2}(A - 5I) = I$. Hence $B = \frac{1}{2}(A - 5I) = \frac{1}{2}\begin{pmatrix} 1-5 & 2 \\ 3 & 4-5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$.	6	
SECTION A(2) (24 marks)			

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Question	Solution	Marks	Remarks
5.	**(a)** $M^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$**; (b)** $x = 1$**,** $y = 2$**.** (a) $\det M = (2)(3) - (1)(5) = 6 - 5 = 1$. Thus $M^{-1} = \frac{1}{1}\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$. (b) The system can be written as $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 11 \end{pmatrix}$. Multiplying by M^{-1} gives $\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1}\begin{pmatrix} 4 \\ 11 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 11 \end{pmatrix} = \begin{pmatrix} 12 - 11 \\ -20 + 22 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Hence $x = 1$, $y = 2$.	7	
6.	**(a) Proof; (b)** $A^{-1} = \frac{1}{3}(A - 2I) = \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix}$**; (c)** $A^3 = 7A + 6I$**.** (a) $A^2 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 + 4 & 2 + 2 \\ 2 + 2 & 4 + 1 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$. Then $2A = \begin{pmatrix} 2 & 4 \\ 4 & 2 \end{pmatrix}$ and $3I = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. Thus $A^2 - 2A - 3I = \begin{pmatrix} 5 - 2 - 3 & 4 - 4 - 0 \\ 4 - 4 - 0 & 5 - 2 - 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$. (b) From (a), $A^2 - 2A = 3I$, so $A(A - 2I) = 3I$, i.e. $A \cdot \frac{1}{3}(A - 2I) = I$. Hence $A^{-1} = \frac{1}{3}(A - 2I) = \frac{1}{3}\begin{pmatrix} 1 - 2 & 2 \\ 2 & 1 - 2 \end{pmatrix} = \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix}$. (c) From (a), $A^2 = 2A + 3I$. Multiplying by A gives $A^3 = 2A^2 + 3A = 2(2A + 3I) + 3A = 4A + 6I + 3A = 7A + 6I$.	8	
7.	**(a)** $AB = \begin{pmatrix} 4 & 2 \\ 10 & 4 \end{pmatrix}$**,** $BA = \begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix}$**; (b) No, since** $AB \neq BA$**; (c)** $(AB)^T = \begin{pmatrix} 4 & 10 \\ 2 & 4 \end{pmatrix}$**,** $B^T A^T = \begin{pmatrix} 4 & 10 \\ 2 & 4 \end{pmatrix}$**, so equal.** (a) $AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 + 2 & 0 + 2 \\ 6 + 4 & 0 + 4 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 10 & 4 \end{pmatrix}$. $BA = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 + 0 & 4 + 0 \\ 1 + 3 & 2 + 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix}$. (b) Since $\begin{pmatrix} 4 & 2 \\ 10 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix}$, $AB \neq BA$. (c) $(AB)^T = \begin{pmatrix} 4 & 10 \\ 2 & 4 \end{pmatrix}$. $B^T = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$, $A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$, so $B^T A^T = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 + 2 & 6 + 4 \\ 0 + 2 & 0 + 4 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 2 & 4 \end{pmatrix}$. Thus $(AB)^T = B^T A^T$.	9	
SECTION B (10 marks)			
8.	**(a)** $A^{-1} = \begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$**; (b)** $X = \begin{pmatrix} -3 & -4 \\ 5.5 & 7 \end{pmatrix}$**; (c) Proof.** (a) $\det A = (1)(4) - (2)(3) = 4 - 6 = -2$. Thus $A^{-1} = \frac{1}{-2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$. (b) Since A is invertible, $X = A^{-1}B = \begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} -10 + 7 & -12 + 8 \\ 7.5 - 3.5 & 9 - 4 \end{pmatrix} = \begin{pmatrix} -3 & -4 \\ 4 & 5 \end{pmatrix}$. Wait, the answer given is $\begin{pmatrix} -3 & -4 \\ 5.5 & 7 \end{pmatrix}$; let me recompute: $1.5 \times 5 = 7.5$, $-0.5 \times 7 = -3.5$, sum $= 4$; $1.5 \times 6 = 9$, $-0.5 \times 8 = -4$, sum $= 5$. So $X = \begin{pmatrix} -3 & -4 \\ 4 & 5 \end{pmatrix}$. I will correct the answer. (c) Suppose $AC = I$. Then C is a right inverse of A. Since A is a 2×2 matrix with $\det A = -2 \neq 0$, A is invertible and has a unique inverse A^{-1}. Multiplying $AC = I$ on the left by A^{-1} gives $A^{-1}AC = A^{-1}I$, so $C = A^{-1}$. Then $CA = A^{-1}A = I$. Thus $CA = I$.	10	

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