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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M2
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Determinants

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. Evaluate the determinant **(3 marks)**

$$\begin{vmatrix} 3 & -2 & 1 \\ 0 & 4 & 5 \\ 2 & 1 & -3 \end{vmatrix}.$$

2. **(3 marks)**

It is given that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5$. Using the properties of determinants, find the value of

$$\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g+3a & h+3b & i+3c \end{vmatrix}.$$

3. Let $A = \begin{pmatrix} 2 & k \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 2 & 5 \end{pmatrix}$, where k is a real constant. **(4 marks)**

(a) Find $\det(AB)$ in terms of k.

(b) Hence find the value of k for which AB is singular.

Answers written in the margins will not be marked.
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4. **(6 marks)**

Let $M = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 2 & 5 & 1 \end{pmatrix}$.

(a) Show that $\det M = 1$.

(b) Without computing M^{-1} directly, write down the value of $\det(M^3)$ and of $\det(2M)$.

(c) The matrix N is obtained from M by interchanging the first and third rows and then multiplying the second row by -4. Find $\det N$.

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SECTION A(2) (24 marks)

5. Consider the system of linear equations in x, y, z: **(7 marks)**

$$\begin{cases} x + 2y - z = 3 \\ 2x + y + z = 0 \\ x - y + kz = 1 \end{cases}$$

where k is a real constant.

(a) Let D be the determinant of the coefficient matrix. Show that $D = 3(k - 2)$.

(b) Find the value of k for which the system does not have a unique solution.

(c) For $k = 3$, solve the system.

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6. Let A and B be 3×3 matrices with $\det A = 4$ and $\det B = -3$. **(8 marks)**

(a) Find $\det(A^2 B^T)$ and $\det(3A^{-1} B)$.

(b) A matrix C satisfies $AC = 2B$. Find $\det C$.

(c) Prove that if P is a 3×3 matrix with $P^2 = P$ and $P \neq O$, then $\det P$ is either 0 or 1.

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7. **(9 marks)**

Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & t \end{pmatrix}$, where t is a real constant.

(a) Find $\det A$ in terms of t.

(b) Find the values of t for which A is singular.

(c) For $t = 1$, find A^{-1} using the adjoint (cofactor) method, and verify that $AA^{-1} = I$.

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SECTION B (10 marks)

8. Let a, b, c be real numbers and define **(10 marks)**

$$D(a,b,c) = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}.$$

(a) By performing the row operations $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, show that

$$D(a,b,c) = (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a+b \\ 0 & 1 & a+c \end{vmatrix}.$$

(b) Hence show that $D(a,b,c) = (b-a)(c-a)(c-b)$.

(c) Deduce that $D(a,b,c) = 0$ if and only if at least two of a, b, c are equal.

(d) Using (b), or otherwise, solve the equation

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & x+1 & (x+1)^2 \\ 1 & x+2 & (x+2)^2 \end{vmatrix} = 6.$$

END OF PAPER

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