MARKING SCHEME

MATHEMATICS Extended Part Module M2 · Paper

Topic practice: Indefinite Integration · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$x^3 - 2\ln\lvert x\rvert + \frac{1}{4}e^{4x} + C$ Integrate term by term: $\int 3x^2\,dx = x^3, \quad \int -\frac{2}{x}\,dx = -2\ln\lvert x\rvert, \quad \int e^{4x}\,dx = \frac{1}{4}e^{4x}$. Hence $\int (3x^2 - \frac{2}{x} + e^{4x})dx = x^3 - 2\ln\lvert x\rvert + \frac{1}{4}e^{4x} + C.$	3	
2.	$\sqrt{x^2+1} + C$ Let $u = x^2 + 1$. Then $du = 2x\,dx$, so $x\,dx = \frac{1}{2}du$. $\int \frac{x}{\sqrt{x^2+1}}\,dx = \int \frac{1}{\sqrt{u}} \cdot \frac{1}{2}\,du = \frac{1}{2}\int u^{-1/2}\,du = u^{1/2} + C.$ Substituting back, the integral is $\sqrt{x^2+1} + C$.	3	
3.	$\frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$ Let $u = x$ and $dv = \cos 2x\,dx$. Then $du = dx$ and $v = \frac{1}{2}\sin 2x$. $\int x\cos 2x\,dx = \frac{x}{2}\sin 2x - \int \frac{1}{2}\sin 2x\,dx = \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C.$	4	
4.	**(a) Proof; (b)** $\frac{x}{2} - \frac{\sin 2x}{4} + C$ (a) Using the identity $\cos 2x = 1 - 2\sin^2 x$, we have $2\sin^2 x = 1 - \cos 2x$, so $\sin^2 x = \frac{1-\cos 2x}{2}$. (b) From (a), $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int (1 - \cos 2x)\,dx = \frac{1}{2}(x - \frac{\sin 2x}{2}) + C = \frac{x}{2} - \frac{\sin 2x}{4} + C.$	6	
SECTION A(2) (24 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
5.	**(a)** $2x+1$**; (b)** x^2+x+C**; (c)** $F(x)=x^2+x+3$ (a) Factorise the numerator: $2x^2+3x+1=(2x+1)(x+1)$. Hence for $x\neq -1$, $$f(x)=\frac{(2x+1)(x+1)}{x+1}=2x+1.$$ (b) $\int f(x)\,dx=\int(2x+1)\,dx=x^2+x+C$. (c) Since $F'(x)=f(x)$, we have $F(x)=x^2+x+C$. Using $F(1)=5$: $$1^2+1+C=5 \implies C=3.$$ Thus $F(x)=x^2+x+3$.	7	
6.	**(a)** $2\sqrt{x}\ln x-4\sqrt{x}+C$**; (b)** $8\ln 2-4$ (a) Let $u=\sqrt{x}$. Then $x=u^2$ and $dx=2u\,du$. Also $\ln x=\ln(u^2)=2\ln u$. $$\int\frac{\ln x}{\sqrt{x}}\,dx=\int\frac{2\ln u}{u}\cdot 2u\,du=4\int\ln u\,du.$$ Using integration by parts, $\int\ln u\,du=u\ln u-u+C$. Hence $$4(u\ln u-u)+C=4\sqrt{x}\ln(\sqrt{x})-4\sqrt{x}+C=2\sqrt{x}\ln x-4\sqrt{x}+C.$$ (b) Evaluate from $x=1$ to $x=4$: $$[2\sqrt{x}\ln x-4\sqrt{x}]_1^4=(4\ln 4-8)-(0-4)=8\ln 2-4.$$	8	
7.	**(a)** $\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+C$**; (b)** $\frac{1}{2}x^3e^{2x}-\frac{3}{4}x^2e^{2x}+\frac{3}{4}xe^{2x}-\frac{3}{8}e^{2x}+C$ (a) Let $u=x^2$, $dv=e^{2x}\,dx$. Then $du=2x\,dx$, $v=\frac{1}{2}e^{2x}$. $$\int x^2e^{2x}\,dx=\frac{1}{2}x^2e^{2x}-\int xe^{2x}\,dx.$$ For $\int xe^{2x}\,dx$, let $u=x$, $dv=e^{2x}\,dx$, giving $du=dx$, $v=\frac{1}{2}e^{2x}$: $$\int xe^{2x}\,dx=\frac{1}{2}xe^{2x}-\frac{1}{2}\int e^{2x}\,dx=\frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+C.$$ Therefore $$\int x^2e^{2x}\,dx=\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+C.$$ (b) Use integration by parts with $u=x^3$, $dv=e^{2x}\,dx$. Then $du=3x^2\,dx$, $v=\frac{1}{2}e^{2x}$. $$\int x^3e^{2x}\,dx=\frac{1}{2}x^3e^{2x}-\frac{3}{2}\int x^2e^{2x}\,dx.$$ Substitute the result from (a): $$=\frac{1}{2}x^3e^{2x}-\frac{3}{2}(\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x})+C=\frac{1}{2}x^3e^{2x}-\frac{3}{4}x^2e^{2x}+\frac{3}{4}xe^{2x}-\frac{3}{8}e^{2x}+C.$$	9	
SECTION B (10 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
8.	**(a) Proof; (b)** $-\cos x + \frac{1}{3}\cos^3 x + C$**; (c)** $\sin x - \frac{1}{3}\sin^3 x + C$ (a) Using $\sin(A + B) = \sin A\cos B + \cos A\sin B$ with $A = 2x$, $B = x$: $\sin 3x = \sin 2x\cos x + \cos 2x\sin x = (2\sin x\cos x)\cos x + (1 - 2\sin^2 x)\sin x.$ $= 2\sin x\cos^2 x + \sin x - 2\sin^3 x = 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x = 3\sin x - 4\sin^3 x.$ (b) Rearranging the identity gives $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$. Hence $\int \sin^3 x\,dx = \frac{1}{4}\int (3\sin x - \sin 3x)\,dx = \frac{1}{4}\left(-3\cos x + \frac{1}{3}\cos 3x\right) + C.$ Using $\cos 3x = 4\cos^3 x - 3\cos x$, this simplifies to $\frac{1}{4}\left(-3\cos x + \frac{4\cos^3 x - 3\cos x}{3}\right) + C = -\cos x + \frac{1}{3}\cos^3 x + C.$ (c) Let $u = \sin x$, so $du = \cos x\,dx$. Then $\int \cos^3 x\,dx = \int \cos^2 x \cdot \cos x\,dx = \int (1 - \sin^2 x)\cos x\,dx = \int (1 - u^2)\,du = u - \frac{u^3}{3} + C.$ Thus $\int \cos^3 x\,dx = \sin x - \frac{1}{3}\sin^3 x + C.$	10	

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