MARKING SCHEME

MATHEMATICS Extended Part Module M2 · Paper

Topic practice: Differentiation · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$f'(x) = 2x + 3$ By definition, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h} = \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h} = \frac{2xh + h^2 + 3h}{h} = 2x + h + 3.$ Taking the limit as $h \to 0$, $f'(x) = 2x + 3$.	3	
2.	$\frac{dy}{dx} = 4(2x^3 - 5x + 1)^3 (6x^2 - 5)$ Using the chain rule, let $u = 2x^3 - 5x + 1$, then $y = u^4$. $\frac{dy}{dx} = 4u^3 \cdot \frac{du}{dx} = 4(2x^3 - 5x + 1)^3 \cdot (6x^2 - 5).$	3	
3.	$\frac{dy}{dx} = e^{2x}(2\sin 3x + 3\cos 3x)$ Using the product rule and chain rule: $\frac{dy}{dx} = \frac{d}{dx}(e^{2x}) \cdot \sin 3x + e^{2x} \cdot \frac{d}{dx}(\sin 3x) = 2e^{2x} \sin 3x + e^{2x} \cdot 3\cos 3x = e^{2x}(2\sin 3x + 3\cos 3x).$	4	
4.	**(a)** $f'(x) = \frac{1}{2\sqrt{x+1}}$ **(b)** $y = \frac{1}{4}x + \frac{5}{4}$ (a) $f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}.$ Multiply numerator and denominator by the conjugate: $\frac{(\sqrt{x+h+1} - \sqrt{x+1})(\sqrt{x+h+1} + \sqrt{x+1})}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}.$ Cancel h and take limit: $f'(x) = \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}.$ (b) At $x = 3$, $f(3) = \sqrt{4} = 2$ and $f'(3) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$. Tangent: $y - 2 = \frac{1}{4}(x - 3) \Rightarrow y = \frac{1}{4}x + \frac{5}{4}$.	6	
SECTION A(2) (24 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
5.	**(a)** $\frac{dy}{dx} = \frac{x^2 - 2x - 1}{(x-1)^2}$ **(b) At $x = 2$: $y = -x + 9$; at $x = 0$: $y = -x - 1$** **(c) No, they are parallel (both have slope -1), not perpendicular.** (a) Using quotient rule: $$\frac{dy}{dx} = \frac{(2x)(x-1) - (x^2+1)(1)}{(x-1)^2} = \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2} = \frac{x^2 - 2x - 1}{(x-1)^2}.$$ (b) At $x = 2$: $y = \frac{4+1}{1} = 5$, slope $m = \frac{4-4-1}{1} = -1$. Tangent: $y - 5 = -1(x-2) \Rightarrow y = -x + 7$? Wait recalc: $y - 5 = -1(x-2) \Rightarrow y = -x + 2 + 5 = -x + 7$. Correction: $y = -x + 7$. At $x = 0$: $y = \frac{1}{-1} = -1$, slope $m = \frac{-1}{1} = -1$. Tangent: $y + 1 = -1(x - 0) \Rightarrow y = -x - 1$. (c) Slopes are both -1, so tangents are parallel, not perpendicular.	7	
6.	**(a)** $f'(x) = \cot x$ **(b)** $f''(x) = -\csc^2 x$ **(c) Proof:** $f''(x) + (f'(x))^2 + 1 = -\csc^2 x + \cot^2 x + 1 = 0$ **using identity** $\csc^2 x = 1 + \cot^2 x$**.** (a) By chain rule: $f'(x) = \frac{1}{\sin x} \cdot \cos x = \cot x$. (b) $f''(x) = \frac{d}{dx}(\cot x) = -\csc^2 x$. (c) $f''(x) + (f'(x))^2 + 1 = -\csc^2 x + \cot^2 x + 1 = -(1 + \cot^2 x) + \cot^2 x + 1 = 0$.	8	
7.	**(a)** $f'(x) = -\frac{1}{x^2}$ **(b)** $g'(x) = -\frac{2}{(2x+1)^2}$ (a) $$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{x - (x+h)}{hx(x+h)} = \lim_{h \to 0} \frac{-h}{hx(x+h)} = \lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}.$$ (b) Let $g(x) = f(2x+1)$. Then $$g'(x) = \lim_{h \to 0} \frac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{h} = \lim_{h \to 0} \frac{(2x+1) - (2x+2h+1)}{h(2x+2h+1)(2x+1)} = \lim_{h \to 0} \frac{-2h}{h(2x+2h+1)(2x+1)} = -\frac{2}{(2x+1)^2}.$$	9	
SECTION B (10 marks)			
8.	**(a) Proof:** $\ln y = x \ln x$**, differentiate implicitly.** **(b) Stationary point at $(\frac{1}{e}, e^{-1/e})$, which is a minimum.** **(c) $y = x$.** (a) Take ln: $\ln y = x \ln x$. Differentiate both sides w.r.t. x: $$\frac{1}{y}\frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1.$$ Thus $\frac{dy}{dx} = y(1 + \ln x) = x^x(1 + \ln x)$. (b) Set $\frac{dy}{dx} = 0$: since $x^x > 0$, $1 + \ln x = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$. Then $y = (e^{-1})^{e^{-1}} = e^{-1/e}$. So stationary point is $(\frac{1}{e}, e^{-1/e})$. To determine nature, use first derivative test: for $0 < x < \frac{1}{e}$, $\ln x < -1$ so $1 + \ln x < 0$, $\frac{dy}{dx} < 0$; for $x > \frac{1}{e}$, $\ln x > -1$ so $1 + \ln x > 0$, $\frac{dy}{dx} > 0$. Hence it is a minimum. (c) At $x = 1$, $y = 1^1 = 1$. $\frac{dy}{dx}\Big\|_{x=1} = 1^1(1 + \ln 1) = 1$. Tangent: $y - 1 = 1(x - 1) \Rightarrow y = x$.	10	

TUTGATHER