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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M2
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Limits

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$, explaining briefly why direct substitution fails. **(3 marks)**

2. Given that $\lim_{x \to 2} f(x) = 5$ and $\lim_{x \to 2} g(x) = -3$, evaluate **(3 marks)**

(a) $\lim_{x \to 2} [2f(x) - g(x)]$;

(b) $\lim_{x \to 2} \frac{f(x)\,g(x)}{f(x) + g(x)}$,

stating the limit theorems you use.

3. The function h is defined by **(4 marks)**

$$h(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & x < 1, \\ k, & x = 1, \\ 2x + 1, & x > 1, \end{cases}$$

where k is a constant. Find the value of k for which h is continuous at $x = 1$.

Answers written in the margins will not be marked.
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4. Consider $\lim_{x \to \infty} \frac{3x^2 - 5x + 2}{2x^2 + 7}$. **(6 marks)**

(a) Divide numerator and denominator by x^2 and evaluate the limit.

(b) Hence describe the behaviour of the graph of $y = \frac{3x^2 - 5x + 2}{2x^2 + 7}$ for large $|x|$, and state the equation of its horizontal asymptote.

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SECTION A(2) (24 marks)

5. (a) Show that $\sqrt{4+h} - 2 = \dfrac{h}{\sqrt{4+h}+2}$ for $h \neq 0$. **(7 marks)**

(b) Hence evaluate $\lim\limits_{h \to 0} \dfrac{\sqrt{4+h}-2}{h}$.

(c) Using the result of (b), evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{4+3x}-2}{x}$.

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6. The function f is defined by **(8 marks)**

$$f(x) = \begin{cases} \dfrac{x^2 + ax - 3}{x - 1}, & x < 1, \\ b, & x = 1, \\ \dfrac{x^2 - 1}{x - 1}, & x > 1, \end{cases}$$

where a and b are constants.

(a) Find $\lim_{x \to 1^-} f(x)$ in terms of a, given that the limit exists.

(b) Find the values of a and b for which f is continuous at $x = 1$.

(c) For these values, determine whether f is differentiable at $x = 1$, justifying your answer.

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7. Let $f(x) = \dfrac{2x^2 + x - 3}{|x - 1|}$. **(9 marks)**

(a) Find $\lim\limits_{x \to 1^+} f(x)$ and $\lim\limits_{x \to 1^-} f(x)$.

(b) Does $\lim\limits_{x \to 1} f(x)$ exist? Justify your answer.

(c) Evaluate $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$.

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SECTION B (10 marks)

8. **(10 marks)**

Let $g(x) = \dfrac{\sqrt{x^2 + px + 9} - 3}{x}$ for $x \neq 0$, where p is a real constant, and define $g(0) = c$.

(a) Show that for $x \neq 0$,

$$g(x) = \frac{x + p}{\sqrt{x^2 + px + 9} + 3}.$$

(b) Find $\lim_{x \to 0} g(x)$ in terms of p.

(c) Find the value of c (in terms of p) for which g is continuous at $x = 0$.

(d) For the case $p = 0$, determine whether g, with the value of c found in (c), is differentiable at $x = 0$. Justify your answer.

END OF PAPER

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