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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M2
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Introduction to e

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. Evaluate $\lim_{n \to \infty}(1 + \frac{3}{n})^n$, giving your answer in terms of e. **(3 marks)**

2. Solve $e^{2x} - 5e^x + 6 = 0$ for real x, giving your answers in exact form. **(3 marks)**

3. Using the binomial expansion of $(1 + \frac{1}{n})^n$, show that **(4 marks)**

$$(1 + \frac{1}{n})^n = 2 + \frac{n-1}{2n} + \frac{(n-1)(n-2)}{6n^2} + \cdots$$

and hence explain briefly why the sequence increases towards e as $n \to \infty$.

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4. Let $f(x) = \ln(\frac{1+x}{1-x})$ for $-1 < x < 1$. **(6 marks)**

(a) Simplify $f(x)$ as a difference of two logarithms.

(b) Show that $f(\frac{2t}{1+t^2}) = 2f(t)$ for $0 < t < 1$.

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SECTION A(2) (24 marks)

5. A sum of \$10 000 is invested at a nominal annual interest rate of 6%. **(7 marks)**

(a) Find the amount after 1 year if interest is compounded n times per year, and write down the limiting amount as $n \to \infty$ (continuous compounding).

(b) Find the least number of times per year interest must be compounded so that the amount after 1 year exceeds \$10 600.

(c) Using continuous compounding, find the time (in years, correct to 3 significant figures) for the investment to double.

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6. Let $f(x) = e^x \sin x$ for $x \geq 0$. **(8 marks)**

(a) Find $f'(x)$ and $f''(x)$.

(b) Solve $f'(x) = 0$ for $x \geq 0$, giving the general solution.

(c) Find the smallest $x > 0$ at which f has a local maximum, and state the corresponding value of $f(x)$ correct to 3 significant figures.

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7. The population P (in thousands) of a bacteria culture after t hours is modelled by **(9 marks)**

$$P(t) = \frac{50}{1 + 4e^{-0.8t}}, \qquad t \geq 0.$$

(a) Find the initial population $P(0)$ and the limiting population as $t \to \infty$.

(b) Show that $P'(t) = kP(1 - \frac{P}{50})$ for some constant k, and state the value of k.

(c) Find the time at which the population is growing fastest, and the population at that instant.

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SECTION B (10 marks)

8. Let $g(x) = e^x - 1 - x - \frac{x^2}{2}$ for all real x. **(10 marks)**

(a) Show that $g'(x) = e^x - 1 - x$ and $g''(x) = e^x - 1$.

(b) Using the signs of g'' and g', prove that $g(x) \geq 0$ for all $x \geq 0$, and deduce that $e^x \geq 1 + x + \frac{x^2}{2}$ for $x \geq 0$.

(c) Hence, or otherwise, show that for $x > 0$,

$$\frac{e^x - 1}{x} > 1 + \frac{x}{2}.$$

(d) Using (c) with $x = \frac{1}{n}$, find $\lim_{n \to \infty} n(e^{1/n} - 1)$ and explain how this limit is consistent with the value of e^x near $x = 0$.

END OF PAPER

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