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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M2
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Mathematical Induction

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. Prove by mathematical induction that for all positive integers n, **(3 marks)**

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2 .$$

2. Prove by mathematical induction that for all positive integers n, **(3 marks)**

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3}.$$

3. Prove by mathematical induction that for all positive integers n, **(4 marks)**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6}.$$

Answers written in the margins will not be marked.

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4. Prove by mathematical induction that for all positive integers n, **(6 marks)**

$$1 + \frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{4n+1}{3n+1}.$$

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SECTION A(2) (24 marks)

5. Prove by mathematical induction that for all positive integers n, **(7 marks)**

$$\sum_{k=1}^{n} (-1)^k k^2 = \frac{(-1)^n n(n+1)}{2}.$$

Hence evaluate $\sum_{k=3}^{333} (-1)^{k+1} k^2$.

Answers written in the margins will not be marked.

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6. Prove by mathematical induction that for all positive integers n, **(8 marks)**

$$\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}.$$

Using this result, evaluate $\sum_{k=1}^{50} \frac{1}{(2k-1)(2k+1)}$ and $\sum_{k=10}^{50} \frac{1}{(2k-1)(2k+1)}$.

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7. Prove by mathematical induction that for all positive integers n, **(9 marks)**

$$\sum_{k=1}^{n} k \cdot 2^k = (n-1)2^{n+1} + 2.$$

Hence find the value of $\sum_{k=1}^{10} k \cdot 2^k$.

Answers written in the margins will not be marked.

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SECTION B (10 marks)

8. (a) Prove by mathematical induction that for all positive integers n, **(10 marks)**

$$\sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)} = \frac{n(n+3)}{4(n+1)(n+2)}.$$

(b) Using the result in (a), evaluate $\sum_{k=1}^{20} \frac{1}{k(k+1)(k+2)}$ and $\sum_{k=5}^{20} \frac{1}{k(k+1)(k+2)}$.

END OF PAPER

Answers written in the margins will not be marked.

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