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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M2
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Odd and Even Functions

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. Determine whether the function $f(x) = x^3 \cos x$ is odd, even, or neither. Justify your answer. **(3 marks)**

2. The graph of $y = g(x)$ is symmetric about the y-axis. Given that $g(2) = 5$ and $g(-3) = 1$, find $g(-2) + g(3)$. **(3 marks)**

3. Let $f(x) = \frac{ax}{x^2 + 1}$, where a is a non-zero constant. Show that f is an odd function. Hence evaluate $f(3) + f(-3)$. **(4 marks)**

Answers written in the margins will not be marked.
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4. The graph of $y = h(x)$ is symmetric about the origin. It is known that $h(x) = x^3 + kx$ for some constant k, and the graph passes through the point $(2, 10)$. **(6 marks)**

(a) Find the value of k.

(b) Hence find $h(-2)$.

(c) Describe the symmetry of the graph of $y = h(x) + 1$.

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SECTION A(2) (24 marks)

5. Let $f(x) = \frac{e^x - e^{-x}}{2}$ and $g(x) = \frac{e^x + e^{-x}}{2}$. **(7 marks)**

(a) Show that f is odd and g is even.

(b) Prove that $g(x)^2 - f(x)^2 = 1$ for all real x.

(c) Hence, or otherwise, determine whether the function $h(x) = f(x)g(x)$ is odd, even, or neither.

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6. The graph of $y = F(x)$ is symmetric about the line $x = 2$. It is given that $F(0) = 3$ and $F(5) = 7$. **(8 marks)**

(a) Find $F(4)$ and $F(-1)$.

(b) If $F(x) = x^2 + bx + c$, find the constants b and c.

(c) Hence describe the symmetry of the graph of $y = F(x - 1)$.

Answers written in the margins will not be marked.

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7. Let $f(x) = \frac{x}{x^2 + 1}$. **(9 marks)**

(a) Show that f is an odd function.

(b) Find $f'(x)$ and determine whether f' is even, odd, or neither.

(c) Using the results of (a) and (b), or otherwise, evaluate $\int_{-1}^{1} f(x)\,dx$ and $\int_{-1}^{1} f'(x)\,dx$.

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SECTION B (10 marks)

8. Let f be a function defined on R. Suppose the graph of $y = f(x)$ is symmetric about the line $x = a$ and also symmetric about the line $x = b$, where $a < b$. **(10 marks)**

(a) Show that $f(2a - x) = f(x)$ and $f(2b - x) = f(x)$ for all real x.

(b) Prove that $f(x + 2(b - a)) = f(x)$ for all real x.

(c) Deduce that f is periodic with period $2(b - a)$.

(d) Give an example of a non-constant function f whose graph is symmetric about both $x = 1$ and $x = 3$, and verify that it satisfies the condition in (b).

END OF PAPER

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