MARKING SCHEME

MATHEMATICS Extended Part Module M2 · Paper

Topic practice: Odd and Even Functions · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**Odd** Compute $f(-x) = (-x)^3\cos(-x) = -x^3\cos x = -f(x)$ since cos is even. Thus f is odd.	3	
2.	**6** Symmetry about the y-axis means g is even, so $g(-x) = g(x)$. Hence $g(-2) = g(2) = 5$ and $g(3) = g(-3) = 1$. Therefore $g(-2) + g(3) = 5 + 1 = 6$.	3	
3.	**0** $f(-x) = \frac{a(-x)}{(-x)^2+1} = -\frac{ax}{x^2+1} = -f(x)$, so f is odd. Therefore $f(3) + f(-3) = f(3) - f(3) = 0$.	4	
4.	**(a)** $k = 1$**; (b)** -10**; (c) symmetric about the point** $(0, 1)$**.** (a) Since $(2, 10)$ lies on the graph, $10 = 2^3 + 2k = 8 + 2k$, so $2k = 2$ and $k = 1$. (b) h is odd (symmetric about origin), so $h(-2) = -h(2) = -10$. (c) Adding 1 shifts the graph vertically by 1 unit. The original symmetry about $(0, 0)$ becomes symmetry about $(0, 1)$.	6	
SECTION A(2) (24 marks)			
5.	**(a) shown; (b) shown; (c) odd.** (a) $f(-x) = \frac{e^{-x}-e^{x}}{2} = -\frac{e^{x}-e^{-x}}{2} = -f(x)$, so f is odd. $g(-x) = \frac{e^{-x}+e^{x}}{2} = g(x)$, so g is even. (b) $g(x)^2 - f(x)^2 = \left(\frac{e^{x}+e^{-x}}{2}\right)^2 - \left(\frac{e^{x}-e^{-x}}{2}\right)^2 = \frac{(e^{2x}+2+e^{-2x}) - (e^{2x}-2+e^{-2x})}{4} = \frac{4}{4} = 1$. (c) $h(-x) = f(-x)g(-x) = (-f(x))(g(x)) = -f(x)g(x) = -h(x)$, so h is odd.	7	

TUTGATHER

Question	Solution	Marks	Remarks
6.	**(a)** $F(4) = 3$**,** $F(-1) = 7$**; (b)** $b = -4$**,** $c = 3$**; (c) symmetric about** $x = 3$**.** (a) Symmetry about $x = 2$ means $F(2 + t) = F(2 - t)$ for all t. Taking $t = 2$: $F(4) = F(0) = 3$. Taking $t = 3$: $F(5) = F(-1)$, so $F(-1) = 7$. (b) Since F is a quadratic symmetric about $x = 2$, its vertex form is $F(x) = (x - 2)^2 + k$. Using $F(0) = 3$: $4 + k = 3 \Rightarrow k = -1$. Thus $F(x) = (x - 2)^2 - 1 = x^2 - 4x + 3$, so $b = -4$, $c = 3$. (c) The graph of $y = F(x - 1)$ is obtained by translating $y = F(x)$ one unit to the right. The axis of symmetry $x = 2$ shifts to $x = 3$.	8	
7.	$\int_{-1}^{1} f(x)\,dx = 0$**,** $\int_{-1}^{1} f'(x)\,dx = 1$ (a) $f(-x) = \frac{-x}{(-x)^2 + 1} = -\frac{x}{x^2 + 1} = -f(x)$, so f is odd. (b) $f'(x) = \frac{(1)(x^2 + 1) - x(2x)}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$. Then $f'(-x) = \frac{1 - (-x)^2}{((-x)^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2} = f'(x)$, so f' is even. (c) Since f is odd, $\int_{-1}^{1} f(x)\,dx = 0$. Since f' is even, $\int_{-1}^{1} f'(x)\,dx = 2\int_{0}^{1} f'(x)\,dx = 2[f(1) - f(0)] = 2(\frac{1}{2} - 0) = 1$.	9	
SECTION B (10 marks)			
8.	**(a) shown; (b) shown; (c) shown; (d) e.g.** $f(x) = \cos(\pi x)$**.** (a) Symmetry about $x = a$ means $f(a + t) = f(a - t)$ for all t. Let $t = a - x$, then $f(2a - x) = f(x)$. Similarly, symmetry about $x = b$ gives $f(2b - x) = f(x)$. (b) Using (a) twice: $f(x + 2(b - a)) = f(2a - (x + 2(b - a)))$? Wait, better: From $f(2a - x) = f(x)$, replace x by $2b - x$: $f(2a - (2b - x)) = f(2b - x)$. But $f(2b - x) = f(x)$, so $f(x + 2(a - b)) = f(x)$. That gives period $2(b - a)$? Actually $2a - 2b + x = x - 2(b - a)$, so $f(x - 2(b - a)) = f(x)$. Since this holds for all x, replacing x by $x + 2(b - a)$ gives $f(x) = f(x + 2(b - a))$. So $f(x + 2(b - a)) = f(x)$. (c) The equation $f(x + 2(b - a)) = f(x)$ for all x means f is periodic with period $2(b - a)$ (or a divisor). (d) Example: $f(x) = \cos(\pi x)$. Symmetric about $x = 1$ because $\cos(\pi(2 - x)) = \cos(2\pi - \pi x) = \cos(\pi x)$. Symmetric about $x = 3$ because $\cos(\pi(6 - x)) = \cos(6\pi - \pi x) = \cos(\pi x)$. Here $a = 1, b = 3$, so $2(b - a) = 4$. Check: $f(x + 4) = \cos(\pi x + 4\pi) = \cos(\pi x) = f(x)$.	10	

TUTGATHER