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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M1
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Confidence Interval

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. A random sample of 25 observations from a normal population with known standard deviation $\sigma = 10$ has sample mean $\bar{x} = 48$. Find a 95% confidence interval for the population mean μ. **(3 marks)**
(Given: $z_{0.025} = 1.96$.)

2. A 90% confidence interval for the mean waiting time (in minutes) at a clinic is found to be (12.4, 17.6). A student says: "The probability that the population mean lies between 12.4 and 17.6 is 0.90." Explain carefully why this statement is not correct, and state the correct interpretation of the 90% confidence level. **(3 marks)**

3. The lifetimes of a large batch of batteries are normally distributed with unknown mean μ and standard deviation 12 hours. A random sample of 36 batteries gives a mean lifetime of 250 hours. Construct a 99% confidence interval for μ. **(4 marks)**
(Given: $z_{0.005} = 2.576$.)

Answers written in the margins will not be marked.
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4. A machine fills bottles with soft drink. The volume (in mL) is normally distributed with standard deviation 5 mL. A random sample of n bottles is taken and a 95% confidence interval for the mean volume μ is constructed. **(6 marks)**

(a) If $n = 100$ and the sample mean is 500 mL, find the width of the 95% confidence interval.

(b) The manager wants the width of the 95% confidence interval to be at most 1 mL. Find the minimum sample size required.

(c) A colleague claims that using a 99% confidence interval instead would allow a smaller sample size for the same width. Comment on this claim.
(Given: $z_{0.025} = 1.96$, $z_{0.005} = 2.576$.)

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SECTION A(2) (24 marks)

5. The weights of packets of tea are normally distributed with mean μ and standard deviation σ, both unknown. A random sample of 16 packets has mean weight 101.2 g and standard deviation 1.6 g. **(7 marks)**

(a) Construct a 95% confidence interval for μ, using the t-distribution.

(b) State one assumption needed for the interval in (a) to be valid.

(c) A second, independent sample of 16 packets has mean weight 99.4 g and standard deviation 1.6 g. The two samples are combined. Construct a 95% confidence interval for μ using the combined sample, assuming the two samples come from the same normal population.
(Given: $t_{15,\ 0.025} = 2.131$, $t_{31,\ 0.025} = 2.040$.)

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6. In a survey of 400 randomly chosen voters in a city, 232 said they support a proposed policy. Let p be the proportion of all voters in the city who support the policy. **(8 marks)**

(a) Find an approximate 95% confidence interval for p.

(b) Interpret the interval found in (a) in the context of the survey.

(c) A newspaper reports: "More than half of the voters support the policy." Using the interval in (a), is this conclusion justified? Explain.

(d) Suppose the survey had used a sample of 1600 voters with the same sample proportion. Without recomputing fully, state how the width of the 95% interval would change, and explain why.
(Given: $z_{0.025} = 1.96$.)

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7. The time (in minutes) taken by students to complete a task is normally distributed with mean μ and standard deviation σ. A random sample of 9 students gives times with mean 22.0 minutes and standard deviation 3.0 minutes. **(9 marks)**

(a) Construct a 90% confidence interval for μ.

(b) A teacher claims that the mean time is 20 minutes. Based on your interval in (a), comment on this claim.

(c) Find the minimum sample size needed so that the *width* of a 90% confidence interval for μ is at most 1.5 minutes, assuming $\sigma = 3.0$ is known.
(Given: $t_{8,\ 0.05} = 1.860$, $z_{0.05} = 1.645$.)

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SECTION B (10 marks)

8. A researcher studies the daily sales (in dollars) of a chain of shops. Sales are modelled as normally distributed with mean μ and standard deviation $\sigma = 40$. **(10 marks)**

(a) A random sample of n days gives a 95% confidence interval for μ of width w. Show that $w = \dfrac{156.8}{\sqrt{n}}$ (to 4 significant figures), and hence find the sample size needed for a width of at most \$10.

(b) The researcher takes a sample of size $n = 64$ and obtains a 95% confidence interval (196.2, 215.8). Find the sample mean.

(c) The researcher now wants a 99% confidence interval of the same width as in (b). Determine the required sample size.

(d) The researcher states: "Because the 99% interval is more reliable, if I use the sample of 64 days the 99% interval will certainly contain μ." Critically assess this statement, referring to the meaning of the confidence level.
(Given: $z_{0.025} = 1.96$, $z_{0.005} = 2.576$.)

END OF PAPER

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