MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Confidence Interval · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$(44.08,\ 51.92)$**, i.e.** 48 ± 3.92 Since the population is normal and σ is known, the 95% confidence interval for μ is $$\bar{x} \pm z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 48 \pm 1.96 \cdot \frac{10}{\sqrt{25}}.$$ Now $\frac{10}{\sqrt{25}} = \frac{10}{5} = 2$, so the margin of error is $1.96 \times 2 = 3.92$. Hence the interval is $$(48 - 3.92,\ 48 + 3.92) = (44.08,\ 51.92).$$	3	
2.	**The population mean is a fixed constant, so it is not a random quantity and has no probability of lying in a fixed interval. The correct interpretation: if the sampling procedure were repeated many times, about 90% of the intervals constructed in this way would contain μ.** The population mean μ is a fixed (unknown) constant, not a random variable. Once the sample has been taken, the interval $(12.4,\ 17.6)$ is fixed, so μ either lies in it or does not — there is no probability attached to it. The 90% refers to the reliability of the *method*: if we repeatedly draw random samples of the same size and construct a 90% confidence interval from each, then in the long run about 90% of these intervals would contain the true mean μ (and about 10% would not).	3	
3.	$(244.85,\ 255.15)$ **hours** Here $n = 36$, $\bar{x} = 250$, $\sigma = 12$ and $z_{0.005} = 2.576$. The standard error is $$\frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{36}} = \frac{12}{6} = 2.$$ The margin of error is $2.576 \times 2 = 5.152$. Hence the 99% confidence interval is $$250 \pm 5.152 = (244.848,\ 255.152) \approx (244.85,\ 255.15) \text{ hours}.$$	4	

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Question	Solution	Marks	Remarks
4.	**(a)** 1.96 **mL (b)** $n \geq 385$ **(minimum** 385**) (c) The claim is wrong: a 99% interval needs a larger z-value, so a larger sample size is required.** (a) Width $= 2z_{0.025}\dfrac{\sigma}{\sqrt{n}} = 2(1.96)\dfrac{5}{\sqrt{100}} = 3.92 \times 0.5 = 1.96$ mL. (b) Require $2(1.96)\dfrac{5}{\sqrt{n}} \leq 1$, i.e. $\dfrac{19.6}{\sqrt{n}} \leq 1 \Rightarrow \sqrt{n} \geq 19.6 \Rightarrow n \geq 384.16.$ Since n is an integer, the minimum sample size is $n = 385$. (c) For a 99% interval the width is $2(2.576)\dfrac{5}{\sqrt{n}}$, which is larger than the 95% width for the same n. To keep the width at most 1 mL we would need $\sqrt{n} \geq 2(2.576)(5) = 25.76$, i.e. $n \geq 663.6$, so $n \geq 664$. Thus a 99% interval requires a *larger* sample size, not a smaller one. The colleague's claim is incorrect: higher confidence comes at the cost of a wider interval (or a larger sample).	6	
SECTION A(2) (24 marks)			
5.	**(a)** (100.35, 102.05) **g (b) The population is normal (weights are normally distributed). (c)** (99.70, 100.90) **g** (a) With $n = 16$, $\bar{x} = 101.2$, $s = 1.6$ and $\nu = 15$ degrees of freedom, the standard error is $\dfrac{s}{\sqrt{n}} = \dfrac{1.6}{4} = 0.4.$ The 95% interval is $101.2 \pm 2.131(0.4) = 101.2 \pm 0.8524 = (100.3476,\ 102.0524) \approx (100.35,\ 102.05)$ g. (b) We assume the packet weights follow a normal distribution (needed because σ is unknown and n is small, so the t-distribution applies). (c) Combined sample: $n = 32$, and the combined mean is $\bar{x}_c = \dfrac{16(101.2) + 16(99.4)}{32} = \dfrac{101.2 + 99.4}{2} = 100.3.$ The pooled variance is $s_p^2 = \dfrac{15(1.6)^2 + 15(1.6)^2}{30} = (1.6)^2 = 2.56, \quad s_p = 1.6.$ With $\nu = 31$ degrees of freedom, the standard error is $\dfrac{1.6}{\sqrt{32}} = 0.28284$, and the interval is $100.3 \pm 2.040(0.28284) = 100.3 \pm 0.5770 = (99.723,\ 100.877) \approx (99.70,\ 100.90)$ g.	7	

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Question	Solution	Marks	Remarks
6.	**(a)** (0.5315, 0.6285) **(b) About 95% of intervals built this way contain p; we are 95% confident p lies between 0.5315 and 0.6285. (c) Yes — the whole interval lies above 0.5. (d) The width is halved (divided by 2), because width $\propto 1/\sqrt{n}$ and $\sqrt{1600}/\sqrt{400} = 2$.** (a) $\hat{p} = \frac{232}{400} = 0.58$, $n = 400$. The standard error is $$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.58 \times 0.42}{400}} = \sqrt{0.000609} \approx 0.024678.$$ The 95% interval is $$0.58 \pm 1.96(0.024678) = 0.58 \pm 0.04837 = (0.5316,\ 0.6284) \approx (0.5315,\ 0.6285).$$ (b) We are 95% confident that the true proportion p of all voters in the city who support the policy lies between about 0.532 and 0.628. Equivalently, if the sampling were repeated many times, about 95% of intervals constructed this way would contain p. (c) Yes. The entire 95% confidence interval (0.5315, 0.6285) lies above 0.5, so at the 95% confidence level the data support the claim that more than half of the voters support the policy. (d) The width of the interval is $2z\sqrt{\hat{p}(1-\hat{p})/n}$, which is proportional to $1/\sqrt{n}$. Since $\sqrt{1600} = 2\sqrt{400}$, the width would be halved (approximately 0.0242 instead of 0.0484), giving a more precise estimate.	8	
7.	**(a)** (20.14, 23.86) **minutes (b)** 20 **lies outside the interval, so the claim is not supported at the 90% level. (c)** $n \geq 44$ **(minimum** 44**)** (a) $n = 9$, $\bar{x} = 22.0$, $s = 3.0$, $\nu = 8$. The standard error is $$\frac{s}{\sqrt{n}} = \frac{3.0}{3} = 1.0.$$ The 90% interval is $$22.0 \pm 1.860(1.0) = 22.0 \pm 1.86 = (20.14,\ 23.86) \text{ minutes.}$$ (b) The value 20 lies below the lower limit 20.14 of the 90% confidence interval. Hence, at the 90% confidence level, the data are not consistent with a mean time of 20 minutes; the teacher's claim is not supported (the sample suggests the mean is higher). (c) With $\sigma = 3.0$ known, the width is $2z_{0.05}\frac{\sigma}{\sqrt{n}} = 2(1.645)\frac{3}{\sqrt{n}} = \frac{9.87}{\sqrt{n}}$. Require $$\frac{9.87}{\sqrt{n}} \leq 1.5 \ \Rightarrow\ \sqrt{n} \geq 6.58 \ \Rightarrow\ n \geq 43.296.$$ Since n is an integer, the minimum sample size is $n = 44$.	9	
SECTION B (10 marks)			

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Question	Solution	Marks	Remarks
8.	**(a)** $w = 156.8/\sqrt{n}$**;** $n \geq 246$ **(minimum** 246**) (b)** $\bar{x} = 206$ **(c)** $n \geq 111$ **(minimum** 111**) (d) The statement is false: a 99% interval still fails to contain** μ **about 1% of the time; higher confidence does not guarantee containment.** (a) The width is $$w = 2z_{0.025}\frac{\sigma}{\sqrt{n}} = 2(1.96)\frac{40}{\sqrt{n}} = \frac{156.8}{\sqrt{n}}.$$ Require $\frac{156.8}{\sqrt{n}} \leq 10$, i.e. $\sqrt{n} \geq 15.68$, so $n \geq 245.86$. Hence the minimum sample size is $n = 246$. (b) The interval is symmetric about the sample mean, so $$\bar{x} = \frac{196.2 + 215.8}{2} = \frac{412.0}{2} = 206.$$ (Check: half-width $= 9.8 = 1.96 \cdot \frac{40}{8}$, consistent with $n = 64$.) (c) The width in (b) is 2(9.8) = 19.6. For a 99% interval of width 19.6: $$2(2.576)\frac{40}{\sqrt{n}} = 19.6 \Rightarrow \frac{206.08}{\sqrt{n}} = 19.6 \Rightarrow \sqrt{n} = 10.514 \Rightarrow n \geq 110.55.$$ So the minimum sample size is $n = 111$. (d) The statement is incorrect. A 99% confidence level means that if the sampling procedure were repeated many times, about 99% of the intervals constructed would contain μ — but about 1% would not. For any single sample (such as the 64 days here), the resulting interval either contains μ or does not; we cannot say it *certainly* contains μ. Increasing the confidence level from 95% to 99% makes the method more reliable on average, but it does not eliminate the possibility of a non-covering interval, so "certainly" is unjustified.	10	

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