MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Sampling Distribution · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**(a)** $\bar{x} = 10$ **(b)** $s \approx 5.43$ (a) $\bar{x} = \frac{4+7+9+12+18}{5} = \frac{50}{5} = 10.$ (b) $\sum (x-\bar{x})^2 = (-6)^2 + (-3)^2 + (-1)^2 + 2^2 + 8^2 = 36 + 9 + 1 + 4 + 64 = 114.$ $s^2 = \frac{114}{5-1} = 28.5$, so $s = \sqrt{28.5} \approx 5.34.$ (Recomputing: $\sqrt{28.5} = 5.3385\ldots \approx 5.34$.)	3	
2.	**Mean** = 150 g**, standard deviation** = 3 g**; by the central limit theorem (large sample,** $n = 64$**).** For a sample of size n from a population with mean μ and standard deviation σ, $E(\bar{X}) = \mu = 150, \qquad \text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{24}{\sqrt{64}} = \frac{24}{8} = 3.$ Since $n = 64$ is large, the central limit theorem states that $\bar{X}$ is approximately normally distributed, so $\bar{X} \approx N(150,\ 3^2)$.	3	
3.	**(a)** $\bar{X} \approx N(80, 2^2)$ **(b)** 0.0668 **(c)** 0.866 (a) $\text{sd}(\bar{X}) = \frac{20}{\sqrt{100}} = 2$, so $\bar{X} \approx N(80,\ 2^2)$. (b) $P(\bar{X} > 83) = P\left(Z > \frac{83-80}{2}\right) = P(Z > 1.5) = 1 - 0.9332 = 0.0668.$ (c) $P(77 < \bar{X} < 83) = P(-1.5 < Z < 1.5) = 2(0.9332) - 1 = 0.8664 \approx 0.866.$	4	
4.	**(a)** $\bar{x} = 3.55$**,** $s \approx 1.55$ **(b) median** = 3.2**, IQR** = 2.05 **(c) median (data skewed by** 6.3**)** (a) $\sum x = 2.1 + 3.4 + 1.8 + 5.2 + 4.0 + 2.6 + 3.0 + 6.3 = 28.4$, so $\bar{x} = \frac{28.4}{8} = 3.55.$ $\sum x^2 = 4.41 + 11.56 + 3.24 + 27.04 + 16.00 + 6.76 + 9.00 + 39.69 = 117.70.$ $s^2 = \frac{117.70 - 8(3.55)^2}{7} = \frac{117.70 - 100.82}{7} = \frac{16.88}{7} = 2.4114\ldots$, so $s \approx 1.55.$ (b) Ordered: 1.8, 2.1, 2.6, 3.0, 3.4, 4.0, 5.2, 6.3. Median $= \frac{3.0+3.4}{2} = 3.2.$ Lower half $\{1.8, 2.1, 2.6, 3.0\}$: $Q_1 = \frac{2.1+2.6}{2} = 2.35$. Upper half $\{3.4, 4.0, 5.2, 6.3\}$: $Q_3 = \frac{4.0+5.2}{2} = 4.6.$ IQR $= 4.6 - 2.35 = 2.25.$ (c) The value 6.3 is an outlier pulling the mean upward, so the median (3.2) is the better measure of centre.	6	
SECTION A(2) (24 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
5.	**(a)** 0.0228 **(b)** 0.904 **(c)** $n = 88$ (a) $\text{sd}(\bar{X}) = \frac{90}{\sqrt{36}} = 15$. $P(X > 1230) = P\left(Z > \frac{1230 - 1200}{15}\right) = P(Z > 2) = 0.0228$. (b) $P(\lvert X - 1200 \rvert < 25) = P\left(\lvert Z \rvert < \frac{25}{15}\right) = P(-1.667 < Z < 1.667) = 2(0.9522) - 1 = 0.9044 \approx 0.904$. (c) We need $P\left(Z > \frac{30}{90/\sqrt{n}}\right) \le 0.05$, i.e. $\frac{\sqrt{n}}{3} \ge 1.6449$. So $\sqrt{n} \ge 4.9347$, $n \ge 24.35$. The smallest integer is $n = 25$. (Check: $n = 25$ gives sd = 18, $z = 1.667$, tail $\approx 0.0478 \le 0.05$.)	7	
6.	**(a) CLT,** $n = 100$ **large (b)** 0.9696 **(c)** $k \approx 501.0$ **(d) small** n**, skewed population** (a) By the central limit theorem, for large n the sampling distribution of $\bar{X}$ is approximately normal regardless of the shape of the population distribution. Here $n = 100$ is large. (b) $\text{sd}(\bar{X}) = \frac{8}{\sqrt{100}} = 0.8$. $P(498.5 < \bar{X} < 501.5) = P\left(\frac{-1.5}{0.8} < Z < \frac{1.5}{0.8}\right) = P(-1.875 < Z < 1.875) = 2(0.9696) - 1 = 0.9392$. (Recomputing with tables: $\Phi(1.875) \approx 0.9696$, giving 0.9392.) (c) $P(\bar{X} > k) = 0.10 \Rightarrow \frac{k - 500}{0.8} = 1.2816 \Rightarrow k = 500 + 1.025 = 501.025 \approx 501.0$ mL. (d) With $n = 4$ the central limit theorem does not apply; the sampling distribution of $\bar{X}$ inherits the skewness of the population, so a normal approximation is unreliable.	8	
7.	**(a)** $\bar{x} = 52$**,** $s \approx 5.48$ **(b) mean** $= 53$**,** $s \approx 5.72$ **(c) batch 1 more consistent** (a) $\sum x = 48 + 52 + 45 + 61 + 50 + 47 + 55 + 49 + 53 + 60 = 520$, so $\bar{x} = 52$. $\sum x^2 = 2304 + 2704 + 2025 + 3721 + 2500 + 2209 + 3025 + 2401 + 2809 + 3600 = 27298$. $s^2 = \frac{27298 - 10(52)^2}{9} = \frac{27298 - 27040}{9} = \frac{258}{9} = 28.667$, so $s \approx 5.35$. (Recomputing: $\sqrt{28.667} = 5.354 \approx 5.35$.) (b) Combined mean $= \frac{10(52) + 10(54)}{20} = 53$. For batch 2: $\sum x^2 = (n-1)s^2 + n\bar{x}^2 = 9(36) + 10(54)^2 = 324 + 29160 = 29484$. Combined $\sum x^2 = 27298 + 29484 = 56782$. $s^2 = \frac{56782 - 20(53)^2}{19} = \frac{56782 - 56180}{19} = \frac{602}{19} = 31.684$, so $s \approx 5.63$. (c) Batch 1 has the smaller standard deviation ($5.35 < 6.0$), so batch 1 has more consistent lifetimes.	9	
SECTION B (10 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
8.	**(a)** $E(\bar{X}) = 2.4$**,** $\text{Var}(\bar{X}) = \frac{0.81}{n}$ **(b)** $n = 78$ **(c)** ≈ 0.163 **(d) claim false (** $\approx 0.951 < 0.99$**)** (a) $E(\bar{X}) = \mu = 2.4$ and $\text{Var}(\bar{X}) = \frac{\sigma^2}{n} = \frac{0.81}{n}$. (b) $\text{sd}(\bar{X}) = \frac{0.9}{\sqrt{n}}$. We need $$P(\lvert Z \rvert < \frac{0.2\sqrt{n}}{0.9}) \geq 0.95 \Rightarrow \frac{0.2\sqrt{n}}{0.9} \geq 1.96.$$ So $\sqrt{n} \geq \frac{1.96 \times 0.9}{0.2} = 8.82$, giving $n \geq 77.79$. The least integer is $n = 78$. (c) With $n = 78$, $\text{sd}(\bar{X}) = \frac{0.9}{\sqrt{78}} = 0.10190 \ldots$. $$P(\bar{X} < 2.3) = P(Z < \frac{2.3 - 2.4}{0.10190}) = P(Z < -0.9814) = 0.1632 \approx 0.163.$$ (d) If n is doubled to 156, $\text{sd}(\bar{X}) = \frac{0.9}{\sqrt{156}} = 0.07206 \ldots$ and $$P(\lvert \bar{X} - 2.4 \rvert < 0.2) = P(\lvert Z \rvert < \frac{0.2}{0.07206}) = P(\lvert Z \rvert < 2.775) = 2(0.99724) - 1 = 0.9945.$$ This is at least 0.99, so the claim is true. (Note: the value $0.9945 \geq 0.99$ confirms the supervisor's claim.)	10	

TUTGATHER