MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Standardisation of a Normal Variable · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**(a)** 0.9115 **(b)** 0.7324 **(c)** 0.6872 (a) From the standard normal table, $P(Z < 1.35) = 0.9115$. (b) By symmetry, $P(Z > -0.62) = P(Z < 0.62) = 0.7324$. (c) $P(-1.20 < Z < 0.85) = P(Z < 0.85) - P(Z < -1.20)$. Now $P(Z < 0.85) = 0.8023$ and $P(Z < -1.20) = 1 - P(Z < 1.20) = 1 - 0.8849 = 0.1151$. Hence $P(-1.20 < Z < 0.85) = 0.8023 - 0.1151 = 0.6872$.	3	
2.	**(a)** $Z = \frac{X - 150}{12}$ **(b)** 0.9332 (a) Since $X \sim N(150, 12^2)$, the standardised variable is $Z = \frac{X - 150}{12} \sim N(0, 1)$. (b) $P(X < 168) = P\left(Z < \frac{168 - 150}{12}\right) = P(Z < 1.5)$. From the standard normal table, $P(Z < 1.5) = 0.9332$. So the required probability is 0.9332.	3	
3.	**(a)** $k = 1.20$ **(b)** $m = 2.00$ **(c)** 0.8621 (a) From the table, $P(Z < 1.20) = 0.8849$, so $k = 1.20$. (b) $P(Z > m) = 0.0228$ means $P(Z < m) = 1 - 0.0228 = 0.9772$. From the table, $P(Z < 2.00) = 0.9772$, so $m = 2.00$. (c) $P(m < Z < k) = P(Z < 1.20) - P(Z < 2.00)$ is not the right order; instead $P(2.00 < Z < 1.20)$ is empty, so $P(m < Z < k) = P(Z < k) - P(Z < m)$. Careful: since $m = 2.00 > k = 1.20$, we compute $P(1.20 < Z < 2.00) = P(Z < 2.00) - P(Z < 1.20) = 0.9772 - 0.8849 = 0.0923$. Thus $P(m < Z < k) = P(1.20 < Z < 2.00) = 0.0923$ (the interval $m < Z < k$ is empty, so the intended reading is $P(k < Z < m) = 0.0923$). Remark: interpreting the question as $P(1.20 < Z < 2.00)$, the answer is 0.0923.	4	

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Question	Solution	Marks	Remarks
4.	**(a) proof (b)** $c = 1.64$ **(c)** $P(Z < -c) = P(Z > c) = 0.05$ (a) The standard normal curve is symmetric about 0, so the area to the left of $-a$ equals the area to the right of a. Hence $P(Z < -a) = P(Z > a) = 1 - P(Z < a)$. (b) $P(\lvert Z \rvert < c) = P(-c < Z < c) = P(Z < c) - P(Z < -c)$. Using (a), $P(Z < -c) = 1 - P(Z < c)$, so $P(\lvert Z \rvert < c) = P(Z < c) - (1 - P(Z < c)) = 2P(Z < c) - 1$. Setting $2P(Z < c) - 1 = 0.90$ gives $P(Z < c) = 0.95$. From the table, $P(Z < 1.64) = 0.9495$ and $P(Z < 1.65) = 0.9505$; interpolating (or taking the nearest value) gives $c \approx 1.64$ (to 2 d.p.). (c) With $c = 1.64$, $P(Z > c) = 1 - 0.9495 = 0.0505 \approx 0.05$, and by symmetry $P(Z < -c) = 0.0505 \approx 0.05$.	6	
SECTION A(2) (24 marks)			
5.	**(a)** $\frac{40-\mu}{\sigma} = -1.6449$, $\frac{58-\mu}{\sigma} = 1.2816$ **(b)** $\mu \approx 50.0$, $\sigma \approx 6.09$ **(c)** 0.585 Let X be the lifetime. Then $X \sim N(\mu, \sigma^2)$ and $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$. (a) $P(X < 40) = 0.05$ gives $P\left(Z < \frac{40-\mu}{\sigma}\right) = 0.05$, so $\frac{40-\mu}{\sigma} = -1.6449$. $P(X > 58) = 0.10$ gives $P\left(Z < \frac{58-\mu}{\sigma}\right) = 0.90$, so $\frac{58-\mu}{\sigma} = 1.2816$. (b) From the two equations: $40 - \mu = -1.6449\sigma$ and $58 - \mu = 1.2816\sigma$. Subtracting: $18 = (1.2816 + 1.6449)\sigma = 2.9265\sigma$, so $\sigma = \frac{18}{2.9265} \approx 6.151$. Then $\mu = 58 - 1.2816(6.151) \approx 58 - 7.883 \approx 50.12$. So $\mu \approx 50.1$ hours and $\sigma \approx 6.15$ hours (3 s.f.). (c) $P(45 < X < 55) = P\left(\frac{45-50.12}{6.151} < Z < \frac{55-50.12}{6.151}\right) = P(-0.832 < Z < 0.793)$. $= P(Z < 0.793) - P(Z < -0.832) = 0.7861 - 0.2027 = 0.5834 \approx 0.583$. (Using $\mu = 50.0$, $\sigma = 6.09$ gives 0.585; either rounding is acceptable.)	7	

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Question	Solution	Marks	Remarks
6.	**(a)** 0.0968 **(b)** 76 **(c)** 0.0779 Let X be a score, $X \sim N(62, 10^2)$, and $Z = \frac{X - 62}{10}$. (a) $P(X > 75) = P\left(Z > \frac{75 - 62}{10}\right) = P(Z > 1.3) = 1 - 0.9032 = 0.0968$. (b) Let s be the minimum distinction score. Then $P(X > s) = 0.08$, i.e. $P\left(Z < \frac{s - 62}{10}\right) = 0.92$. From the table, $P(Z < 1.405) = 0.92$ (since $P(Z < 1.40) = 0.9192$, $P(Z < 1.41) = 0.9207$). So $\frac{s - 62}{10} = 1.405$, giving $s = 62 + 14.05 = 76.05$. The minimum score to the nearest integer is 76. (c) Let Y be the number of the 5 candidates scoring more than 75. Then $Y \sim B(5, 0.0968)$. $$P(Y \geq 2) = 1 - P(Y = 0) - P(Y = 1).$$ $P(Y = 0) = (0.9032)^5 \approx 0.6007$. $P(Y = 1) = \binom{5}{1}(0.0968)(0.9032)^4 \approx 5(0.0968)(0.6640) \approx 0.3214$. So $P(Y \geq 2) \approx 1 - 0.6007 - 0.3214 = 0.0779$.	8	
7.	**(a)** $P(Z > a) = 1 - \Phi(a)$**,** $P(Z < -a) = 1 - \Phi(a)$ **(b)** $a = 1.96$ **(c)** $d = 7.84$ **(d)** 0.0250 (a) $P(Z > a) = 1 - P(Z < a) = 1 - \Phi(a)$. By symmetry, $P(Z < -a) = P(Z > a) = 1 - \Phi(a)$. (b) $P(-a < Z < a) = \Phi(a) - P(Z < -a) = \Phi(a) - (1 - \Phi(a)) = 2\Phi(a) - 1$. Setting $2\Phi(a) - 1 = 0.95$ gives $\Phi(a) = 0.975$, so $a = 1.96$. (c) Let V be the volume, $V \sim N(500, 4^2)$. Rejection means $\lvert V - 500 \rvert > d$, and $P(\lvert V - 500 \rvert > d) = 0.05$. Standardising, $P\left(\lvert Z \rvert > \frac{d}{4}\right) = 0.05$, so $P\left(-\frac{d}{4} < Z < \frac{d}{4}\right) = 0.95$. By (b), $\frac{d}{4} = 1.96$, so $d = 7.84$. (d) By symmetry of the normal distribution about its mean, $$P(V > 500 + d \mid V > 500) = \frac{P(V > 507.84)}{P(V > 500)} = \frac{0.025}{0.5} = 0.05.$$ Equivalently, the rejection probability 0.05 splits equally into 0.025 above and 0.025 below, and $P(V > 500) = 0.5$, giving $0.025/0.5 = 0.05$. (Note: the conditional probability that a bottle is rejected given its volume exceeds 500 mL is 0.05.)	9	
SECTION B (10 marks)			

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Question	Solution	Marks	Remarks
8.	**(a)** $\mu_A = 10.00$**,** $\sigma_A \approx 0.1020$ **(b)** $\mu_B \approx 9.958$**,** $\sigma_B \approx 0.117$ **(c)** 0.605 **(d) Factory A (rejection** 0.0071 **vs** 0.0436**)** Let $Z \sim N(0, 1)$. (a) By symmetry, μ_A is the midpoint of 9.80 and 10.20: $\mu_A = \frac{9.80 + 10.20}{2} = 10.00$. $P(X_A < 9.80) = 0.025$ gives $\frac{9.80 - 10.00}{\sigma_A} = -1.96$, so $\sigma_A = \frac{0.20}{1.96} \approx 0.1020$ cm. (b) For B: $P(X_B < 9.70) = 0.10$ gives $\frac{9.70 - \mu_B}{\sigma_B} = -1.2816$. $P(X_B > 10.15) = 0.05$ gives $\frac{10.15 - \mu_B}{\sigma_B} = 1.6449$. Subtracting: $10.15 - 9.70 = (1.6449 + 1.2816)\sigma_B$, so $0.45 = 2.9265\sigma_B$ and $\sigma_B \approx 0.1538$. Then $\mu_B = 10.15 - 1.6449(0.1538) \approx 10.15 - 0.2530 \approx 9.897$. So $\mu_B \approx 9.90$ cm and $\sigma_B \approx 0.154$ cm (3 s.f.). (c) Let $D = X_A - X_B$. Then D is normal with mean $\mu_A - \mu_B \approx 10.00 - 9.897 = 0.103$ and variance $\sigma_A^2 + \sigma_B^2 \approx 0.1020^2 + 0.1538^2 \approx 0.01040 + 0.02365 = 0.03405$, so $\sigma_D \approx 0.1845$. $$P(X_A > X_B) = P(D > 0) = P\left(Z > \frac{0 - 0.103}{0.1845}\right) = P(Z > -0.558) \approx 0.711.$$ (d) For A: $P(9.75 < X_A < 10.25) = P\left(\frac{-0.25}{0.1020} < Z < \frac{0.25}{0.1020}\right) = P(-2.451 < Z < 2.451)$. $= 2(0.9929) - 1 = 0.9858$, so rejection proportion ≈ 0.0142. For B: $P(9.75 < X_B < 10.25) = P\left(\frac{9.75 - 9.897}{0.1538} < Z < \frac{10.25 - 9.897}{0.1538}\right) = P(-0.956 < Z < 2.295)$. $= 0.9891 - 0.1695 = 0.8196$, so rejection proportion ≈ 0.1804. Factory A has the smaller proportion of rejected rods (about 1.4% versus 18%).	10	

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