MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Basic Definition and Properties of Normal Distribution · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$k = \frac{1}{2}$ For f to be a probability density function, the total area under the curve must be 1: $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 kx\,dx = k[\frac{x^2}{2}]_0^2 = k \cdot \frac{4}{2} = 2k.$ Set $2k = 1$, giving $k = \frac{1}{2}$.	3	
2.	0.683 **(3 s.f.)** Standardise: $P(42 < X < 58) = P(\frac{42-50}{8} < Z < \frac{58-50}{8}) = P(-1 < Z < 1).$ Using the standard normal table, $P(Z < 1) \approx 0.8413$, so $P(-1 < Z < 1) = 2(0.8413) - 1 = 0.6826 \approx 0.683$ (3 s.f.).	3	
3.	**Proportion** = 0.5**;** $P(138 < W < 162) \approx 0.683$ **(3 s.f.)** The normal curve is symmetric about the mean $\mu = 150$. Since the total area under the curve is 1, the area to the right of the mean is 0.5. Hence the proportion of apples weighing more than 150 g is 0.5. For $138 < W < 162$: $P(138 < W < 162) = P(\frac{138-150}{12} < Z < \frac{162-150}{12}) = P(-1 < Z < 1) \approx 0.683$ (3 s.f.).	4	

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Question	Solution	Marks	Remarks
4.	**(a)** $c = \frac{3}{4}$ **(b)** $\frac{11}{16}$ (a) The total area under f must be 1: $$\int_{-1}^{1} c(1 - x^2)\,dx = c[x - \frac{x^3}{3}]_{-1}^{1} = c(\frac{2}{3} - (-\frac{2}{3})) = \frac{4c}{3}.$$ Set $\frac{4c}{3} = 1$, giving $c = \frac{3}{4}$. (b) Using $c = \frac{3}{4}$: $$P(-\frac{1}{2} < X < \frac{1}{2}) = \frac{3}{4}\int_{-1/2}^{1/2} (1 - x^2)\,dx = \frac{3}{4}[x - \frac{x^3}{3}]_{-1/2}^{1/2}.$$ Evaluate the bracket: $$(\frac{1}{2} - \frac{1}{24}) - (-\frac{1}{2} + \frac{1}{24}) = \frac{11}{24} + \frac{11}{24} = \frac{11}{12}.$$ Thus $P(-\frac{1}{2} < X < \frac{1}{2}) = \frac{3}{4} \cdot \frac{11}{12} = \frac{11}{16}$.	6	
SECTION A(2) (24 marks)			
5.	**(a)** 0.0912 **(3 s.f.) (b)** $k \approx 953$ **hours** Let T be the lifetime of a bulb, so $T \sim N(1200, 150^2)$. (a) $P(T > 1400) = P(Z > \frac{1400 - 1200}{150}) = P(Z > 1.3333) = 1 - 0.9088 = 0.0912$ (3 s.f.). (b) We require $P(T < k) = 0.05$. Standardising, $$P(Z < \frac{k - 1200}{150}) = 0.05.$$ From the standard normal table, $P(Z < -1.6449) \approx 0.05$. Hence $$\frac{k - 1200}{150} = -1.6449 \quad \Rightarrow \quad k = 1200 - 150(1.6449) = 953.27.$$ To the nearest hour, $k \approx 953$ hours.	7	
6.	**(a)** $d \approx 12.82$ **(b)** 0.1 Let $H \sim N(170, 10^2)$. (a) Since the interval is symmetric about the mean, $P(170 - d < H < 170 + d) = 0.8$. By symmetry, the area in each tail is $\frac{1 - 0.8}{2} = 0.1$. Thus $$P(H > 170 + d) = 0.1 \quad \Rightarrow \quad P(Z > \frac{d}{10}) = 0.1.$$ From the standard normal table, $P(Z > 1.2816) \approx 0.1$, so $$\frac{d}{10} = 1.2816 \quad \Rightarrow \quad d = 12.816 \approx 12.82 \text{ (2 d.p.)}.$$ (b) From part (a), $P(H > b) = P(H > 170 + d) = 0.1$.	8	

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Question	Solution	Marks	Remarks
7.	**(a)** $k = \frac{1}{4}$ **(b)** $\frac{3}{4}$ **(c)** 2 (a) The total area under f is 1: $\int_0^2 kx\,dx + \int_2^4 k(4-x)\,dx = k[\frac{x^2}{2}]_0^2 + k[4x - \frac{x^2}{2}]_2^4 = k(2) + k(8-6) = 4k.$ Set $4k = 1$, giving $k = \frac{1}{4}$. (b) Using $k = \frac{1}{4}$: $P(1 < X < 3) = \int_1^2 \frac{x}{4}\,dx + \int_2^3 \frac{4-x}{4}\,dx = \frac{1}{4}[\frac{x^2}{2}]_1^2 + \frac{1}{4}[4x - \frac{x^2}{2}]_2^3.$ First integral: $\frac{1}{4}(2 - \frac{1}{2}) = \frac{3}{8}$. Second integral: $\frac{1}{4}((12 - \frac{9}{2}) - (8-2)) = \frac{1}{4}(\frac{15}{2} - 6) = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$. Thus $P(1 < X < 3) = \frac{3}{8} + \frac{3}{8} = \frac{3}{4}$. (c) The median m satisfies $P(X \le m) = 0.5$. Since the distribution is symmetric about $x = 2$ (the two triangular halves are mirror images), the median is $m = 2$. Check: $P(X \le 2) = \int_0^2 \frac{x}{4}\,dx = \frac{1}{4} \cdot 2 = 0.5$.	9	
SECTION B (10 marks)			
8.	**(a) shown (b)** $\mu \approx 64.06$, $\sigma \approx 18.77$ **(c)** 0.397 **(3 s.f.)** Let X be the test score, so $X \sim N(\mu, \sigma^2)$. (a) Given $P(X < 40) = 0.1$: $P(Z < \frac{40-\mu}{\sigma}) = 0.1.$ From the standard normal table, $P(Z < -1.2816) \approx 0.1$, so $\frac{40-\mu}{\sigma} = -1.2816 \quad \Rightarrow \quad \mu - 40 = 1.2816\sigma. \quad (1)$ Given $P(X > 80) = 0.2$, so $P(X < 80) = 0.8$: $P(Z < \frac{80-\mu}{\sigma}) = 0.8.$ From the table, $P(Z < 0.8416) \approx 0.8$, so $\frac{80-\mu}{\sigma} = 0.8416 \quad \Rightarrow \quad 80 - \mu = 0.8416\sigma. \quad (2)$ (b) Adding (1) and (2): $40 = (1.2816 + 0.8416)\sigma = 2.1232\sigma \quad \Rightarrow \quad \sigma = \frac{40}{2.1232} \approx 18.77$ (2 d.p.). Substitute into (1): $\mu = 40 + 1.2816(18.77) \approx 64.06$ (2 d.p.). (c) Using $\mu = 64.06$ and $\sigma = 18.77$: $P(50 < X < 70) = P(\frac{50-64.06}{18.77} < Z < \frac{70-64.06}{18.77}) = P(-0.749 < Z < 0.316).$ From the standard normal table, $P(Z < 0.316) \approx 0.6240$ and $P(Z < -0.749) \approx 0.2270$. Hence $P(50 < X < 70) \approx 0.6240 - 0.2270 = 0.397$ (3 s.f.).	10	

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