MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Applications of Binomial and Poisson Distributions · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**0.114** Let X be the number of traffic accidents per day. Then $X \sim \text{Po}(3.2)$. The required probability is $P(X = 5) = \dfrac{e^{-3.2}\,3.2^5}{5!} \approx 0.11397 \approx 0.114$ (3 s.f.)	3	
2.	**0.485** Let Y be the number of times a '6' appears in 10 rolls. Then $Y \sim B(10, \frac{1}{6})$. The required probability is $P(Y \le 1) = P(Y = 0) + P(Y = 1) = \left(\frac{5}{6}\right)^{10} + \binom{10}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{9}$. Numerically, $\left(\frac{5}{6}\right)^{10} \approx 0.1615, \quad 10 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{9} \approx 0.3230,$ so $P(Y \le 1) \approx 0.4845 \approx 0.485$ (3 s.f.).	3	
3.	**0.113** Let X be the number of emails received in 2 hours. Since the rate is 8 per hour, $X \sim \text{Po}(16)$. Then $P(X > 20) = 1 - P(X \le 20) = 1 - \sum_{k=0}^{20} \dfrac{e^{-16}16^k}{k!}$. Using a calculator, $P(X \le 20) \approx 0.8867$, so $P(X > 20) \approx 0.1133 \approx 0.113$ (3 s.f.).	4	
4.	**(a) 0.189 (b) 0.642 (c) 1** Let X be the number of defective bulbs in the sample. Then $X \sim B(20, 0.05)$. (a) $P(X = 2) = \binom{20}{2}(0.05)^2(0.95)^{18} \approx 0.1887 \approx 0.189$ (3 s.f.). (b) $P(X \ge 1) = 1 - P(X = 0) = 1 - (0.95)^{20} \approx 1 - 0.3585 = 0.6415 \approx 0.642$ (3 s.f.). (c) $E(X) = np = 20 \times 0.05 = 1$.	6	
SECTION A(2) (24 marks)			

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Question	Solution	Marks	Remarks
5.	**(a) 0.195 (b) 0.594 (c) 0.0354** Let X be the number of calls in a given minute. Then $X \sim \text{Po}(4)$. (a) $P(X = 3) = \frac{e^{-4}4^3}{3!} \approx 0.1954 \approx 0.195$ (3 s.f.). (b) In 30 seconds, the mean number of calls is $4 \times 0.5 = 2$. Let $Y \sim \text{Po}(2)$. Then $P(Y \geq 2) = 1 - P(Y = 0) - P(Y = 1) = 1 - e^{-2} - 2e^{-2} = 1 - 3e^{-2} \approx 0.59399 \approx 0.594$ (3 s.f.). (c) Since the numbers of calls in disjoint time intervals are independent, $P(X = 2) \cdot P(X = 1) = (\frac{e^{-4}4^2}{2!})(\frac{e^{-4}4^1}{1!}) = e^{-8} \cdot 8 \cdot 4 = 32e^{-8} \approx 0.03537 \approx 0.0354$ (3 s.f.).	7	
6.	**(a) 0.206 (b) 0.278 (c) mean 4.5, sd 1.77** Let X be the number of people with blood type A in the sample. Then $X \sim B(15, 0.3)$. (a) $P(X = 5) = \binom{15}{5}(0.3)^5(0.7)^{10} \approx 0.2061 \approx 0.206$ (3 s.f.). (b) $P(X \geq 6) = 1 - P(X \leq 5) = 1 - \sum_{k=0}^{5}\binom{15}{k}(0.3)^k(0.7)^{15-k} \approx 1 - 0.7216 = 0.2784 \approx 0.278$ (3 s.f.). (c) $E(X) = np = 15 \times 0.3 = 4.5$. The standard deviation is $\sqrt{np(1-p)} = \sqrt{15 \times 0.3 \times 0.7} = \sqrt{3.15} \approx 1.7748 \approx 1.77$ (3 s.f.).	8	
7.	**(a) 0.180 (b) 0.982 (c) 0.0324** Let X be the number of particles emitted in a given second. Then $X \sim \text{Po}(2)$. (a) $P(X = 3) = \frac{e^{-2}2^3}{3!} = \frac{8e^{-2}}{6} = \frac{4}{3}e^{-2} \approx 0.1804 \approx 0.180$ (3 s.f.). (b) In 2 seconds, the mean number of particles is $2 \times 2 = 4$. Let $Y \sim \text{Po}(4)$. Then $P(Y \geq 1) = 1 - P(Y = 0) = 1 - e^{-4} \approx 0.98168 \approx 0.982$ (3 s.f.). (c) Since the numbers of particles in disjoint intervals are independent, $P(X = 2) \cdot P(X = 3) = (\frac{e^{-2}2^2}{2!})(\frac{e^{-2}2^3}{3!}) = (2e^{-2})(\frac{4}{3}e^{-2}) = \frac{8}{3}e^{-4} \approx 0.03244 \approx 0.0324$ (3 s.f.).	9	
SECTION B (10 marks)			
8.	**(a) 0.175 (b) 0.713 (c) 0.0146 (d) 9** Let X be the number of patients arriving in a given hour. Then $X \sim \text{Po}(5)$. (a) $P(X = 4) = \frac{e^{-5}5^4}{4!} \approx 0.1755 \approx 0.175$ (3 s.f.). (b) In 30 minutes, the mean number of patients is $5 \times 0.5 = 2.5$. Let $Y \sim \text{Po}(2.5)$. Then $P(Y \geq 2) = 1 - P(Y = 0) - P(Y = 1) = 1 - e^{-2.5} - 2.5e^{-2.5} = 1 - 3.5e^{-2.5} \approx 0.7127 \approx 0.713$ (3 s.f.). (c) Since the numbers of arrivals in disjoint hours are independent, $P(X = 3) \cdot P(X = 2) = (\frac{e^{-5}5^3}{3!})(\frac{e^{-5}5^2}{2!}) = (\frac{125}{6}e^{-5})(\frac{25}{2}e^{-5}) = \frac{3125}{12}e^{-10} \approx 0.01457 \approx 0.0146$ (3 s.f.). (d) We need the smallest integer n such that $P(X \leq n) \geq 0.95$. Using a calculator, $P(X \leq 8) \approx 0.9319, \quad P(X \leq 9) \approx 0.9682.$ Thus the minimum number of beds required is 9.	10	

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