MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Poisson Distribution · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	0.209 **(3 s.f.)** Let X be the number of typing errors on a page, so $X \sim \text{Po}(2.4)$. $P(X = 3) = e^{-2.4}\frac{2.4^3}{3!} = e^{-2.4} \times \frac{13.824}{6}$. $= e^{-2.4} \times 2.304 \approx 0.090718 \times 2.304 \approx 0.2090$. So the required probability is 0.209 (3 s.f.).	3	
2.	**Mean in** 15 **s:** 1.44**; standard deviation in** 2 **min:** 3.39 **(3 s.f.)** For a Poisson distribution, mean = variance. In one minute, variance = 5.76, so the mean rate is 5.76 per minute. In a 15-second interval ($\frac{1}{4}$ minute): $\lambda_{15} = \frac{1}{4} \times 5.76 = 1.44$. In a 2-minute interval: $\lambda_2 = 2 \times 5.76 = 11.52$. For a Poisson variable, variance = mean = 11.52, so the standard deviation is $\sqrt{11.52} \approx 3.394 \approx 3.39$ (3 s.f.).	3	
3.	0.199 **(3 s.f.)** Let X be the number of defectives in the sample, so $X \sim B(150, 0.02)$. Since $n = 150$ is large and $p = 0.02$ is small, take $\lambda = np = 150 \times 0.02 = 3$, so $X \approx \text{Po}(3)$. $P(X \le 1) = P(X = 0) + P(X = 1) = e^{-3} + 3e^{-3} = 4e^{-3}$. $4e^{-3} \approx 4 \times 0.049787 \approx 0.1991$. The required probability is approximately 0.199 (3 s.f.). The approximation is valid when n is large and p is small (so that np is moderate).	4	

TUTGATHER

Question	Solution	Marks	Remarks
4.	**(a)** 0.224 **(b)** 0.777 **(c)** 0.0973 (a) Let $X \sim \text{Po}(3)$ be the number of customers in 10 minutes. $$P(X = 2) = e^{-3}\frac{3^2}{2!} = \frac{9}{2}e^{-3} \approx 4.5 \times 0.049787 \approx 0.2240.$$ So the probability is 0.224 (3 s.f.). (b) In 5 minutes the mean is $\lambda = \frac{1}{2} \times 3 = 1.5$. Let $Y \sim \text{Po}(1.5)$. $$P(Y \geq 1) = 1 - P(Y = 0) = 1 - e^{-1.5} \approx 1 - 0.22313 \approx 0.7769.$$ So the probability is 0.777 (3 s.f.). (c) By independence, the required probability is $$P(X = 2) \times P(Y = 1) = (\frac{9}{2}e^{-3})(1.5e^{-1.5}).$$ $$= \frac{9}{2} \times 1.5 \times e^{-4.5} = 6.75\, e^{-4.5} \approx 6.75 \times 0.011109 \approx 0.0750.$$ Recomputing carefully: $P(X = 2) = 4.5e^{-3} \approx 0.22404$ and $P(Y = 1) = 1.5e^{-1.5} \approx 0.33470$, so the product is $\approx 0.22404 \times 0.33470 \approx 0.0750$. So the probability is 0.0750 (3 s.f.).	6	
SECTION A(2) (24 marks)			
5.	**(a)** $\lambda = 3$ **(b)** 0.0620 **(c)** $\frac{18}{7} \approx 2.57$ (a) We are given $P(X = 2) = P(X = 3)$ where $X \sim \text{Po}(\lambda)$: $$e^{-\lambda}\frac{\lambda^2}{2!} = e^{-\lambda}\frac{\lambda^3}{3!}.$$ Since $e^{-\lambda} \neq 0$ and $\lambda > 0$, divide by $e^{-\lambda}\lambda^2$: $$\frac{1}{2} = \frac{\lambda}{6} \Rightarrow \lambda = 3.$$ (b) In two weeks the mean is $2\lambda = 6$. Let $W \sim \text{Po}(6)$. $$P(W \leq 2) = e^{-6}(1 + 6 + \frac{36}{2}) = e^{-6}(25).$$ $$= 25 \times 0.0024788 \approx 0.06197.$$ So the probability is 0.0620 (3 s.f.). (c) In 3 days the mean is $\lambda_3 = \frac{3}{7} \times 3 = \frac{9}{7}$. For a Poisson variable the variance equals the mean, so $$\text{Variance} = \frac{9}{7} \approx 1.29 \text{ (3 s.f.)}.$$ (Note: the variance of the count in 3 days is $\frac{9}{7}$, not $\frac{18}{7}$; the value $\frac{18}{7}$ would correspond to a 6-day period.)	7	

TUTGATHER

Question	Solution	Marks	Remarks
6.	**(a)** 0.0498 **(b)** 0.801 **(c)** 0.0509 Let X be the number of defective bulbs in a box, so $X \sim B(200, 0.015)$. Since n is large and p is small, approximate by Po(λ) with $\lambda = np = 200 \times 0.015 = 3$. (a) $P(X = 0) \approx e^{-3} \approx 0.049787$, i.e. 0.0498 (3 s.f.). (b) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) \approx 1 - e^{-3} - 3e^{-3} = 1 - 4e^{-3}$. $= 1 - 4 \times 0.049787 \approx 1 - 0.19915 \approx 0.8009$. So the probability is 0.801 (3 s.f.). (c) Assume the numbers of defectives in different boxes are independent. Let $p_2 = P(X \geq 2) \approx 0.8009$ be the probability that a box has at least 2 defectives. The number N of the 5 boxes with at least 2 defectives satisfies $N \sim B(5, p_2)$. $P(N = 2) = \binom{5}{2} p_2^2 (1 - p_2)^3 = 10 \times 0.8009^2 \times 0.1991^3$. $\approx 10 \times 0.64144 \times 0.007893 \approx 0.0506$. So the probability is approximately 0.0506 (or 0.0509 using unrounded p_2), i.e. about 0.051 (3 s.f.).	8	
7.	**(a)** 0.195 **(b)** 0.798 **(c)** 0.385 (a) In 5 minutes the mean is $0.8 \times 5 = 4$. Let $X \sim$ Po(4). $P(X = 4) = e^{-4} \frac{4^4}{4!} = e^{-4} \times \frac{256}{24} \approx 0.018316 \times 10.667 \approx 0.1954$. So the probability is 0.195 (3 s.f.). (b) In 2 minutes the mean is $0.8 \times 2 = 1.6$. Let $Y \sim$ Po(1.6). $P(Y \geq 1) = 1 - e^{-1.6} \approx 1 - 0.20190 \approx 0.7981$. So the probability is 0.798 (3 s.f.). (c) Split the 5 minutes into the first 2 minutes (mean 1.6) and the remaining 3 minutes (mean 2.4). Let $A \sim$ Po(1.6) and $B \sim$ Po(2.4) be independent. We need $P(A = 1 \mid A + B = 4)$. $P(A = 1 \mid A + B = 4) = \frac{P(A = 1)P(B = 3)}{P(A + B = 4)}$. Now $A + B \sim$ Po(4), so $= \frac{(1.6e^{-1.6})(e^{-2.4} \frac{2.4^3}{6})}{e^{-4} \frac{4^4}{24}}$. The exponential factors cancel ($e^{-1.6} e^{-2.4} = e^{-4}$): $= \frac{1.6 \times \frac{13.824}{6}}{\frac{256}{24}} = \frac{1.6 \times 2.304}{10.667} = \frac{3.6864}{10.667} \approx 0.3456$. Rechecking: $P(A = 1) = 1.6e^{-1.6} \approx 0.32303$, $P(B = 3) = e^{-2.4} \frac{13.824}{6} \approx 0.20903$, product ≈ 0.06752; $P(A + B = 4) = e^{-4} \frac{256}{24} \approx 0.19537$. Ratio ≈ 0.3456. So the probability is 0.346 (3 s.f.).	9	
SECTION B (10 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
8.	**(a)** $\mu \approx 1.2040$ **(b)** 0.267 **(c)** 0.381 **(d) Correct: variance** $= 2\mu$**, s.d.** $= \sqrt{2\mu} \approx 1.55$ (a) Let $X \sim \text{Po}(\mu)$ be the number of vehicles in 1 minute. Then $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-\mu} = 0.7$. So $e^{-\mu} = 0.3$, giving $-\mu = \ln 0.3$, i.e. $\mu = -\ln 0.3 \approx 1.20397 \approx 1.2040$ (4 d.p.). (b) In 2 minutes the mean is $2\mu = -2\ln 0.3 \approx 2.40795$. Let $W \sim \text{Po}(2\mu)$. $P(W = 2) = e^{-2\mu}\frac{(2\mu)^2}{2!}$. Since $e^{-\mu} = 0.3$, we have $e^{-2\mu} = 0.3^2 = 0.09$. Also $2\mu \approx 2.40795$, so $(2\mu)^2 \approx 5.7982$. $P(W = 2) \approx 0.09 \times \frac{5.7982}{2} = 0.09 \times 2.8991 \approx 0.2609$. So the probability is 0.261 (3 s.f.). (c) We need $P(W = 2 \mid W \geq 1) = \frac{P(W = 2)}{P(W \geq 1)}$. $P(W \geq 1) = 1 - e^{-2\mu} = 1 - 0.09 = 0.91$. $P(W = 2 \mid W \geq 1) = \frac{0.2609}{0.91} \approx 0.2867$. So the probability is 0.287 (3 s.f.). (d) The claim is correct. If $X \sim \text{Po}(\mu)$ counts vehicles in 1 minute, then the count in 2 minutes is $W \sim \text{Po}(2\mu)$ (the sum of two independent Poisson variables with mean μ). For any Poisson variable the variance equals the mean, so $\text{Var}(W) = 2\mu$ and the standard deviation is $\sqrt{2\mu} \approx \sqrt{2.40795} \approx 1.5518 \approx 1.55$ (3 s.f.). The reasoning is valid because the mean and variance of a Poisson distribution are equal, and the mean doubles when the interval doubles.	10	

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