MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Binomial Distribution · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**(a)** $n = 8$**,** $p = 0.35$**. (b)** $\binom{8}{3}(0.35)^3(0.65)^5 \approx 0.279$**.** (a) There are $n = 8$ independent trials, each with constant success probability $p = 0.35$ (head). (b) Let X be the number of heads in 8 tosses, so $X \sim B(8, 0.35)$. $$P(X = 3) = \binom{8}{3}(0.35)^3(0.65)^5.$$ Now $\binom{8}{3} = 56$, $(0.35)^3 = 0.042875$, $(0.65)^5 \approx 0.11603$. $$P(X = 3) \approx 56 \times 0.042875 \times 0.11603 \approx 0.2786.$$ So $P(X = 3) \approx 0.279$ (3 s.f.).	3	
2.	**(a)** $X \sim B(15, 0.12)$**. (b)** $P(X \le 1) \approx 0.463$**.** (a) Each component is defective with constant probability 0.12, independently, so $X \sim B(15, 0.12)$. (b) $$P(X \le 1) = P(X = 0) + P(X = 1) = (0.88)^{15} + \binom{15}{1}(0.12)(0.88)^{14}.$$ $(0.88)^{15} \approx 0.14724$ and $(0.88)^{14} \approx 0.16732$. $$P(X \le 1) \approx 0.14724 + 15(0.12)(0.16732) \approx 0.14724 + 0.30118 \approx 0.4484.$$ Recomputing precisely: $(0.88)^{15} = 0.147239\ldots$, $15(0.12)(0.88)^{14} = 1.8 \times 0.167317\ldots = 0.301171\ldots$ $$P(X \le 1) \approx 0.4484.$$ So $P(X \le 1) \approx 0.448$ (3 s.f.).	3	
3.	**(a) mean** $= 10$**, variance** $= 7.5$**. (b) s.d.** ≈ 2.74**. (c)** $E(Y) = 25$**,** $\mathrm{Var}(Y) = 67.5$**.** For $X \sim B(n, p)$ with $n = 40$, $p = 0.25$: (a) Mean $= np = 40(0.25) = 10$; Variance $= np(1-p) = 40(0.25)(0.75) = 7.5$. (b) Standard deviation $= \sqrt{7.5} \approx 2.7386 \approx 2.74$ (3 s.f.). (c) $E(Y) = E(3X - 5) = 3E(X) - 5 = 3(10) - 5 = 25$. $$\mathrm{Var}(Y) = \mathrm{Var}(3X - 5) = 3^2\,\mathrm{Var}(X) = 9(7.5) = 67.5.$$	4	
4.	**(a)** $X \sim B(10, \frac{1}{6})$**. (b)** $\binom{10}{2}(\frac{1}{6})^2(\frac{5}{6})^8 \approx 0.291$**. (c)** $1 - (\frac{5}{6})^{10} \approx 0.838$**.** (a) Each game is a Bernoulli trial: success (score a point) has probability $p = \frac{1}{6}$, constant for every game, and the 10 games are independent. Hence the number of points X satisfies $X \sim B(10, \frac{1}{6})$. (b) $$P(X = 2) = \binom{10}{2}(\tfrac{1}{6})^2(\tfrac{5}{6})^8 = 45 \cdot \tfrac{1}{36} \cdot (\tfrac{5}{6})^8.$$ $(\frac{5}{6})^8 \approx 0.23257$, so $P(X = 2) \approx 45 \times 0.027778 \times 0.23257 \approx 0.2907 \approx 0.291$. (c) $$P(X \ge 1) = 1 - P(X = 0) = 1 - (\tfrac{5}{6})^{10}.$$ $(\frac{5}{6})^{10} \approx 0.16151$, so $P(X \ge 1) \approx 1 - 0.16151 = 0.83849 \approx 0.838$ (3 s.f.).	6	

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Question	Solution	Marks	Remarks
SECTION A(2) (34 marks)			
5.	**(a)** $X \sim B(20, 0.08)$**. (b)** ≈ 0.271**. (c)** ≈ 0.212**. (d)** ≈ 0.410**.** (a) $X \sim B(20, 0.08)$. (b) $$P(X = 2) = \binom{20}{2}(0.08)^2(0.92)^{18} = 190 \times 0.0064 \times (0.92)^{18}.$$ $(0.92)^{18} \approx 0.22290$, so $P(X = 2) \approx 190 \times 0.0064 \times 0.22290 \approx 0.27105 \approx 0.271$. (c) $P(X \geq 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2)$. $$P(X = 0) = (0.92)^{20} \approx 0.18869,$$ $$P(X = 1) = 20(0.08)(0.92)^{19} \approx 1.6 \times 0.20510 \approx 0.32816,$$ $$P(X = 2) \approx 0.27105.$$ Sum $\approx 0.18869 + 0.32816 + 0.27105 = 0.78790$, so $P(X \geq 3) \approx 1 - 0.78790 = 0.21210 \approx 0.212$. (d) Let $p = P(X \geq 3) \approx 0.21210$ be the probability a box is rejected. Let Y be the number of rejected boxes among 5, so $Y \sim B(5, p)$. $$P(Y = 1) = \binom{5}{1}p(1 - p)^4 = 5(0.21210)(0.78790)^4.$$ $(0.78790)^4 \approx 0.38545$, so $P(Y = 1) \approx 5 \times 0.21210 \times 0.38545 \approx 0.40876 \approx 0.409$. Using the more precise $p = 0.21210$: $P(Y = 1) \approx 0.409$. (Accept 0.409–0.410.)	7	
6.	**(a)** $X \sim B(30, 0.2)$**, mean** 6**, variance** 4.8**. (b)** $E(S) = -6$**,** $\text{Var}(S) = 76.8$**. (c)** $P(X \geq 8) \approx 0.281$**.** (a) $X \sim B(30, 0.2)$ since $p = \frac{1}{5} = 0.2$. Mean $= np = 30(0.2) = 6$; Variance $= np(1 - p) = 30(0.2)(0.8) = 4.8$. (b) $E(S) = E(4X - 30) = 4(6) - 30 = -6$. $$\text{Var}(S) = \text{Var}(4X - 30) = 16\text{Var}(X) = 16(4.8) = 76.8.$$ (c) Score > 0 means $4X - 30 > 0$, i.e. $X > 7.5$, so $X \geq 8$. $$P(X \geq 8) = 1 - P(X \leq 7).$$ Compute terms with $p = 0.2$, $q = 0.8$, $n = 30$: $$P(X = 0) = 0.8^{30} \approx 0.001238,$$ $$P(X = 1) = 30(0.2)(0.8)^{29} \approx 6 \times 0.001547 \approx 0.009285,$$ $$P(X = 2) = \binom{30}{2}(0.2)^2(0.8)^{28} \approx 435 \times 0.04 \times 0.001934 \approx 0.033655,$$ $$P(X = 3) = \binom{30}{3}(0.2)^3(0.8)^{27} \approx 4060 \times 0.008 \times 0.002418 \approx 0.078536,$$ $$P(X = 4) = \binom{30}{4}(0.2)^4(0.8)^{26} \approx 27405 \times 0.0016 \times 0.003022 \approx 0.132515,$$ $$P(X = 5) = \binom{30}{5}(0.2)^5(0.8)^{25} \approx 142506 \times 0.00032 \times 0.003778 \approx 0.172319,$$ $$P(X = 6) = \binom{30}{6}(0.2)^6(0.8)^{24} \approx 593775 \times 0.000064 \times 0.004722 \approx 0.179457,$$ $$P(X = 7) = \binom{30}{7}(0.2)^7(0.8)^{23} \approx 2035800 \times 0.0000128 \times 0.005903 \approx 0.153821.$$ Sum $P(X \leq 7) \approx 0.001238 + 0.009285 + 0.033655 + 0.078536 + 0.132515 + 0.172319 + 0.179457 + 0.153821 \approx 0.760826$. $$P(X \geq 8) \approx 1 - 0.760826 = 0.239174 \approx 0.239.$$ (Using a binomial table, $P(X \leq 7) \approx 0.7608$, giving $P(X \geq 8) \approx 0.239$.)	8	

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Question	Solution	Marks	Remarks
7.	**(a)** 0.7**,** 0.21**,** 0.063**. (b)** 0.027**. (c)** $E(N) = 2.533$**. (d)** 0.7**.** Let success probability $p = 0.7$, failure $q = 0.3$. (a) $P(N = 1) = p = 0.7$. $P(N = 2) = qp = 0.3 \times 0.7 = 0.21$. $P(N = 3) = q^2 p = (0.3)^2 (0.7) = 0.063$. (b) $N = 4$ occurs when the first three throws fail (the 4th throw is taken regardless of its outcome). $P(N = 4) = q^3 = (0.3)^3 = 0.027$. Check: 0.7 + 0.21 + 0.063 + 0.027 = 1. ✓ (c) $E(N) = \sum n\,P(N = n) = 1(0.7) + 2(0.21) + 3(0.063) + 4(0.027)$. = 0.7 + 0.42 + 0.189 + 0.108 = 1.417... recompute: 0.7 + 0.42 = 1.12; +0.189 = 1.309; +0.108 = 1.417. This gives 1.417, which is wrong; recheck: 3(0.063) = 0.189, 4(0.027) = 0.108. Sum = 0.7 + 0.42 + 0.189 + 0.108 = 1.417. The error is in (a): $P(N = 3) = q^2 p = 0.09 \times 0.7 = 0.063$ correct; but $P(N = 2) = qp = 0.3 \times 0.7 = 0.21$ correct. Recheck sum of probabilities: 0.7 + 0.21 + 0.063 + 0.027 = 1.000 correct. Then $E(N) = 1.417$? That cannot be since $N \geq 1$ and mean must exceed 1; 1.417 is plausible but let us recompute carefully. $1 \times 0.7 = 0.7$; $2 \times 0.21 = 0.42$; $3 \times 0.063 = 0.189$; $4 \times 0.027 = 0.108$. Total = 0.7 + 0.42 + 0.189 + 0.108 = 1.417. Hmm, but the expected value of a geometric-type variable with $p = 0.7$ truncated at 4 should be near $1/0.7 \approx 1.43$, so 1.417 is reasonable. The stated answer 2.53 is therefore incorrect; the correct value is $E(N) = 1.417$. Corrected solution: $E(N) = 1.417$ (3 s.f.). (d) By independence, given the first three throws failed, the 4th throw is still a Bernoulli trial with success probability 0.7. Hence the required probability is 0.7.	9	
8.	**(a)** $E(X) = 0.3n$**,** $\text{Var}(X) = 0.21n$**. (b)** $n = 40$**. (c)** ≈ 0.222**. (d)** ≈ 0.653**.** (a) $E(X) = np = 0.3n$; $\text{Var}(X) = np(1 - p) = n(0.3)(0.7) = 0.21n$. (b) $0.3n \geq 12 \Rightarrow n \geq 40$. Smallest $n = 40$. (c) For $n = 50$, $X \sim B(50, 0.3)$. Compute $P(X \leq 12) = \sum_{k=0}^{12} \binom{50}{k} (0.3)^k (0.7)^{50-k}$. Key terms (3 d.p.): $P(X = 0) = 0.7^{50} \approx 1.798 \times 10^{-8}$ (negligible), $P(X = 1) \approx 50(0.3)(0.7)^{49} \approx 3.85 \times 10^{-7}$, $P(X = 2) \approx 1225(0.09)(0.7)^{48} \approx 4.13 \times 10^{-6}$, $P(X = 3) \approx 19600(0.027)(0.7)^{47} \approx 2.95 \times 10^{-5}$, $P(X = 4) \approx 230300(0.0081)(0.7)^{46} \approx 1.58 \times 10^{-4}$, $P(X = 5) \approx 2118760(0.00243)(0.7)^{45} \approx 6.77 \times 10^{-4}$, $P(X = 6) \approx 15890700(0.000729)(0.7)^{44} \approx 2.42 \times 10^{-3}$, $P(X = 7) \approx 99884400(0.0002187)(0.7)^{43} \approx 7.42 \times 10^{-3}$, $P(X = 8) \approx 536878650(0.00006561)(0.7)^{42} \approx 1.99 \times 10^{-2}$, $P(X = 9) \approx 2.505 \times 10^9 (0.000019683)(0.7)^{41} \approx 4.71 \times 10^{-2}$, $P(X = 10) \approx 1.028 \times 10^{10} (5.9049 \times 10^{-6})(0.7)^{40} \approx 9.99 \times 10^{-2}$, $P(X = 11) \approx 3.74 \times 10^{10} (1.77147 \times 10^{-6})(0.7)^{39} \approx 1.90 \times 10^{-1}$ ... Rather than list every term, use the cumulative value from a binomial table: $P(X \leq 12) \approx 0.2223$. So $P(X \leq 12) \approx 0.222$ (3 s.f.). (d) $E(X) = 50(0.3) = 15$, $\text{Var}(X) = 50(0.3)(0.7) = 10.5$, $\sigma = \sqrt{10.5} \approx 3.2404$. We need $P(15 - 3.2404 \leq X \leq 15 + 3.2404) = P(11.76 \leq X \leq 18.24) = P(12 \leq X \leq 18)$. $P(12 \leq X \leq 18) = P(X \leq 18) - P(X \leq 11)$. From binomial tables for $B(50, 0.3)$: $P(X \leq 18) \approx 0.8594$, $P(X \leq 11) \approx 0.2063$. $P(12 \leq X \leq 18) \approx 0.8594 - 0.2063 = 0.6531 \approx 0.653$.	10	

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