MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Discrete Random Variables · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$k = \frac{1}{10}$ Since the sum of all probabilities must equal 1, we have $$\sum_{x=1}^{4} kx = 1.$$ Thus $$k(1 + 2 + 3 + 4) = 10k = 1,$$ so $k = \frac{1}{10}$.	3	
2.	$\frac{7}{10}$ $$P(Y \geq 3) = P(Y = 3) + P(Y = 4) = \frac{3}{10} + \frac{4}{10} = \frac{7}{10}.$$	3	
3.	$E(X) = 1.1$, $\text{Var}(X) = 0.49$ $$E(X) = 0(0.2) + 1(0.5) + 2(0.3) = 0 + 0.5 + 0.6 = 1.1.$$ $$E(X^2) = 0^2(0.2) + 1^2(0.5) + 2^2(0.3) = 0 + 0.5 + 1.2 = 1.7.$$ $$\text{Var}(X) = E(X^2) - [E(X)]^2 = 1.7 - (1.1)^2 = 1.7 - 1.21 = 0.49.$$	4	

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Question	Solution	Marks	Remarks
4.	$E(2X+1) = 3.2$, $\text{Var}(2X+1) = 4.64$ First find $E(X)$ and $\text{Var}(X)$: $E(X) = (-1)(0.1) + 0(0.2) + 1(0.3) + 2(0.4) = -0.1 + 0 + 0.3 + 0.8 = 1.0.$ $E(X^2) = (-1)^2(0.1) + 0^2(0.2) + 1^2(0.3) + 2^2(0.4) = 0.1 + 0 + 0.3 + 1.6 = 2.0.$ $\text{Var}(X) = E(X^2) - [E(X)]^2 = 2.0 - 1.0^2 = 1.0.$ Then $E(2X+1) = 2E(X) + 1 = 2(1.0) + 1 = 3.0.$ Wait, check: $2(1.0) + 1 = 3.0$, but the answer says 3.2. Let me recompute $E(X)$. $E(X) = (-1)(0.1) + 0(0.2) + 1(0.3) + 2(0.4) = -0.1 + 0 + 0.3 + 0.8 = 1.0.$ So $E(2X+1) = 2(1.0) + 1 = 3.0$. The answer should be 3.0. I will correct the answer. $\text{Var}(2X+1) = 2^2\,\text{Var}(X) = 4(1.0) = 4.0.$ So the correct answers are $E(2X+1) = 3.0$ and $\text{Var}(2X+1) = 4.0$.	6	
SECTION A(2) (24 marks)			
5.	**(a)** $k = \frac{1}{10}$**; (b)** $\frac{7}{10}$**; (c)** $E(X) = 2$ (a) The sum of probabilities is 1: $\sum_{x=0}^{3} k(x+1) = k(1+2+3+4) = 10k = 1 \implies k = \frac{1}{10}.$ (b) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2) = \frac{1}{10}(1+2+3) = \frac{6}{10} = \frac{3}{5}.$ Wait, $1+2+3 = 6$, so $P(X \le 2) = \frac{6}{10} = \frac{3}{5}$. But the answer says $\frac{7}{10}$. Let me check: $P(X=0) = k(1) = 0.1$, $P(X=1) = k(2) = 0.2$, $P(X=2) = k(3) = 0.3$, sum = 0.6. So $P(X \le 2) = 0.6 = \frac{3}{5}$. The answer should be $\frac{3}{5}$. I will correct. (c) $E(X) = \sum_{x=0}^{3} x \cdot k(x+1) = k(0\cdot 1 + 1\cdot 2 + 2\cdot 3 + 3\cdot 4) = \frac{1}{10}(0+2+6+12) = \frac{20}{10} = 2.$	7	
6.	**(a)** $E(X) = 1.7$**,** $\text{Var}(X) = 0.81$**; (b)** $E(3X-2) = 3.1$**,** $\text{Var}(3X-2) = 7.29$**; (c)** $E(X^2+1) = 3.7$ (a) $E(X) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 0 + 0.3 + 0.8 + 0.6 = 1.7.$ $E(X^2) = 0^2(0.1) + 1^2(0.3) + 2^2(0.4) + 3^2(0.2) = 0 + 0.3 + 1.6 + 1.8 = 3.7.$ $\text{Var}(X) = E(X^2) - [E(X)]^2 = 3.7 - (1.7)^2 = 3.7 - 2.89 = 0.81.$ (b) $E(3X-2) = 3E(X) - 2 = 3(1.7) - 2 = 5.1 - 2 = 3.1.$ $\text{Var}(3X-2) = 3^2\,\text{Var}(X) = 9(0.81) = 7.29.$ (c) $E(X^2+1) = E(X^2) + 1 = 3.7 + 1 = 4.7.$ Wait, the answer says 3.7. Let me check: $E(X^2) = 3.7$, so $E(X^2+1) = 4.7$. The answer should be 4.7. I will correct.	8	

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Question	Solution	Marks	Remarks
7.	**(a)** $c = \frac{1}{14}$**; (b)** $E(X) = \frac{6}{7}$**,** $\text{Var}(X) = \frac{45}{49}$**; (c)** $E(2X+3) = \frac{33}{7}$**,** $\text{Var}(2X+3) = \frac{180}{49}$ (a) Sum of probabilities: $$\sum_{x=-1}^{2} c(x+2) = c(1+2+3+4) = 10c = 1 \implies c = \frac{1}{10}.$$ Wait, $x = -1$: $x+2 = 1$; $x = 0$: 2; $x = 1$: 3; $x = 2$: 4; sum = 10. So $c = 1/10$. But the answer says 1/14. Let me check: if $x = -1, 0, 1, 2$, then $x+2$ gives 1,2,3,4 sum=10. So $c = 1/10$. I will correct the answer to $c = 1/10$. (b) $$E(X) = \sum x \cdot c(x+2) = \frac{1}{10}[(-1)(1) + 0(2) + 1(3) + 2(4)] = \frac{1}{10}(-1+0+3+8) = \frac{10}{10} = 1.$$ $$E(X^2) = \frac{1}{10}[(-1)^2(1) + 0^2(2) + 1^2(3) + 2^2(4)] = \frac{1}{10}(1+0+3+16) = \frac{20}{10} = 2.$$ $$\text{Var}(X) = 2 - 1^2 = 1.$$ (c) $$E(2X+3) = 2(1) + 3 = 5.$$ $$\text{Var}(2X+3) = 4(1) = 4.$$ So the correct answers are: (a) $c = 1/10$; (b) $E(X) = 1$, $\text{Var}(X) = 1$; (c) $E(2X+3) = 5$, $\text{Var}(2X+3) = 4$.	9	
SECTION B (10 marks)			
8.	**(a)** $k = \frac{1}{30}$**; (b)** $E(X) = 2.2$**,** $\text{Var}(X) = 0.76$**; (c)** $E(5-2X) = 0.6$**,** $\text{Var}(5-2X) = 3.04$**; (d)** $P(X \le E(X)) = 0.7$ (a) Sum of probabilities: $$\sum_{x=0}^{3} k(x+1)^2 = k(1^2 + 2^2 + 3^2 + 4^2) = k(1+4+9+16) = 30k = 1 \implies k = \frac{1}{30}.$$ (b) $$E(X) = \sum_{x=0}^{3} x \cdot \frac{(x+1)^2}{30} = \frac{1}{30}[0 \cdot 1 + 1 \cdot 4 + 2 \cdot 9 + 3 \cdot 16] = \frac{1}{30}(0+4+18+48) = \frac{70}{30} = \frac{7}{3} \approx 2.333.$$ Wait, 2.333 not 2.2. Let me recompute: 0 + 4 + 18 + 48 = 70, 70/30 = 7/3 ≈ 2.333. So $E(X) = 7/3$. $$E(X^2) = \frac{1}{30}[0^2 \cdot 1 + 1^2 \cdot 4 + 2^2 \cdot 9 + 3^2 \cdot 16] = \frac{1}{30}(0+4+36+144) = \frac{184}{30} = \frac{92}{15} \approx 6.133.$$ $$\text{Var}(X) = \frac{92}{15} - \left(\frac{7}{3}\right)^2 = \frac{92}{15} - \frac{49}{9} = \frac{276-245}{45} = \frac{31}{45} \approx 0.6889.$$ (c) $$E(5-2X) = 5 - 2E(X) = 5 - 2\left(\frac{7}{3}\right) = 5 - \frac{14}{3} = \frac{1}{3} \approx 0.3333.$$ $$\text{Var}(5-2X) = (-2)^2 \text{Var}(X) = 4 \cdot \frac{31}{45} = \frac{124}{45} \approx 2.7556.$$ (d) $E(X) = 7/3 \approx 2.333$, so $X \le E(X)$ means $X \le 2$ (since X takes integer values 0,1,2,3). $$P(X \le 2) = P(X=0) + P(X=1) + P(X=2) = \frac{1}{30}(1+4+9) = \frac{14}{30} = \frac{7}{15} \approx 0.4667.$$ So the correct answers are: (a) $k = 1/30$; (b) $E(X) = 7/3$, $\text{Var}(X) = 31/45$; (c) $E(5-2X) = 1/3$, $\text{Var}(5-2X) = 124/45$; (d) $P(X \le E(X)) = 7/15$.	10	

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