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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

MATHEMATICS Extended Part Module M1
PAPER

Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Approximation of Definite Integrals using the Trapezoidal Rule

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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SECTION A(1) (16 marks)

1. Use the trapezoidal rule with 4 sub-intervals to estimate $\int_0^2 \sqrt{1+x^3}\,dx$. Give your answer correct to 3 significant figures. **(3 marks)**

2. The curve $y = f(x)$ is concave upward on $[a, b]$. State whether the trapezoidal rule with n sub-intervals gives an over-estimate or an under-estimate of $\int_a^b f(x)\,dx$. Justify your answer briefly. **(3 marks)**

3. Let $f(x) = e^{-x^2}$. It is known that $f''(x) = (4x^2 - 2)e^{-x^2}$. Use the trapezoidal rule with 4 sub-intervals to estimate $\int_0^2 f(x)\,dx$, and determine whether this estimate is an over-estimate or an under-estimate. Justify your answer. **(4 marks)**

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4. The table below gives selected values of a continuous function f. **(6 marks)**

x	**0**	**1**	**2**	**3**	**4**	**5**	**6**
$f(x)$	2.0	3.1	4.5	5.2	5.8	6.1	6.3

(a) Use the trapezoidal rule with 6 sub-intervals to estimate $\int_0^6 f(x)\,dx$.

(b) If it is known that $f''(x) < 0$ for all x in $[0, 6]$, determine whether the estimate in (a) is an over-estimate or an under-estimate. Justify your answer.

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SECTION A(2) (24 marks)

5. Let $I = \int_1^3 \ln(x^2 + 1)\, dx$. **(7 marks)**

(a) Use the trapezoidal rule with 4 sub-intervals to estimate I. Give your answer correct to 4 decimal places.

(b) Show that $\frac{d^2}{dx^2}\ln(x^2 + 1) = \frac{2(1 - x^2)}{(x^2 + 1)^2}$.

(c) Determine whether the estimate in (a) is an over-estimate or an under-estimate. Justify your answer.

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6. Let $f(x) = \frac{1}{1 + x^2}$ and $I = \int_0^1 f(x)\,dx$. **(8 marks)**

(a) Use the trapezoidal rule with 4 sub-intervals to estimate I. Give your answer correct to 4 decimal places.

(b) Find $f''(x)$ and determine the intervals on which f is concave upward and concave downward in $[0, 1]$.

(c) By considering the concavity of f, determine whether the estimate in (a) is an over-estimate or an under-estimate. Justify your answer.

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7. The velocity v (in m/s) of a particle moving along a straight line is measured at various times t (in seconds) as shown in the table. **(9 marks)**

t	**0**	**2**	**4**	**6**	**8**	**10**
v	0	4.2	7.1	8.8	9.5	9.9

(a) Use the trapezoidal rule with 5 sub-intervals to estimate the distance travelled by the particle from $t = 0$ to $t = 10$.

(b) It is known that the acceleration $a = \frac{dv}{dt}$ is positive and decreasing on $[0, 10]$. Determine whether the estimate in (a) is an over-estimate or an under-estimate of the actual distance. Justify your answer.

(c) If the acceleration is actually constant, what would be the exact distance? Compare with your estimate in (a).

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SECTION B (10 marks)

8. Let $f(x) = e^{-x^2}$ and $I = \int_0^1 f(x)\,dx$. **(10 marks)**

(a) Use the trapezoidal rule with n sub-intervals to estimate I. Let T_n be this estimate. Show that $T_n = \frac{1}{2n}[1 + e^{-1} + 2\sum_{k=1}^{n-1} e^{-(k/n)^2}]$.

(b) It is known that $f''(x) = (4x^2 - 2)e^{-x^2}$. Determine the intervals on which f is concave upward and concave downward in $[0, 1]$.

(c) By considering the concavity of f, determine whether T_n is an over-estimate or an under-estimate of I for $n = 2$ and for $n = 4$. Justify your answers.

(d) The error of the trapezoidal rule is given by $E_n = I - T_n = -\frac{1}{12n^2}f''(\xi)$ for some $\xi \in (0, 1)$ if f'' does not change sign on $[0, 1]$. However, f'' changes sign on $[0, 1]$. By splitting the interval at the inflection point, or otherwise, show that $|E_n| \leq \frac{1}{6n^2}$.

END OF PAPER

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