MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Indefinite Integration and its Applications · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$2x^3 + \frac{2}{x^2} + 5x + C$ Rewrite $\frac{4}{x^3} = 4x^{-3}$. Then $\int (6x^2 - 4x^{-3} + 5)dx = 6 \cdot \frac{x^3}{3} - 4 \cdot \frac{x^{-2}}{-2} + 5x + C = 2x^3 + \frac{2}{x^2} + 5x + C.$ Check by differentiating: $\frac{d}{dx}(2x^3 + 2x^{-2} + 5x) = 6x^2 - 4x^{-3} + 5$, which matches the integrand.	3	
2.	$\frac{(x^2+1)^6}{6} + C$ Let $u = x^2 + 1$, so $\frac{du}{dx} = 2x$, i.e. $du = 2x\,dx$. Then $\int 2x(x^2+1)^5\,dx = \int u^5\,du = \frac{u^6}{6} + C = \frac{(x^2+1)^6}{6} + C.$ Differentiating gives $\frac{6(x^2+1)^5 \cdot 2x}{6} = 2x(x^2+1)^5$, as required.	3	
3.	$\frac{3}{2}e^{2x} - \frac{1}{2}\ln(2x+7) + C$ Integrate term by term. For $3e^{2x}$, since $\frac{d}{dx}e^{2x} = 2e^{2x}$, we have $\int 3e^{2x}\,dx = \frac{3}{2}e^{2x}$. For $\frac{1}{2x+7}$, note $\frac{d}{dx}\ln(2x+7) = \frac{2}{2x+7}$, so $\int \frac{1}{2x+7}\,dx = \frac{1}{2}\ln(2x+7)$. Hence $\int (3e^{2x} - \frac{1}{2x+7})dx = \frac{3}{2}e^{2x} - \frac{1}{2}\ln(2x+7) + C.$	4	
4.	**(a)** $y = x^3 - 4x^2 + 2x + 5$ **(b)** 1 (a) Integrating the slope function, $y = \int (3x^2 - 8x + 2)dx = x^3 - 4x^2 + 2x + C.$ Since $(1, 4)$ lies on Γ: $4 = 1 - 4 + 2 + C$, so $C = 5$. Hence $y = x^3 - 4x^2 + 2x + 5.$ (b) At $x = 2$: $y = 8 - 16 + 4 + 5 = 1$.	6	
SECTION A(2) (24 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
5.	$I = \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$ (a) Let $u = x + 3$. Then $x = u - 3$ and $du = dx$. Hence $$I = \int (u-3)u^{1/2}\,du = \int (u^{3/2} - 3u^{1/2})du.$$ (b) Integrating, $$I = \frac{u^{5/2}}{5/2} - 3\cdot\frac{u^{3/2}}{3/2} + C = \frac{2}{5}u^{5/2} - 2u^{3/2} + C.$$ Replacing u by $x + 3$: $$I = \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C.$$ (c) Differentiating, $\frac{d}{dx}[\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2}] = (x+3)^{3/2} - 3(x+3)^{1/2} = (x+3)^{1/2}[(x+3) - 3] = x\sqrt{x+3}$, which is the original integrand, so the answer is correct.	7	
6.	**(a)** $T = 12e^{-t/2} + 4\ln(t+1) + 68$ **(b)** 76.8°C **(c)** $T \to \infty$ **(increases without bound)** (a) Integrating term by term: $\int -6e^{-t/2}\,dt = 12e^{-t/2}$ (since $\frac{d}{dt}e^{-t/2} = -\frac{1}{2}e^{-t/2}$) and $\int \frac{4}{t+1}\,dt = 4\ln(t+1)$. Hence $$T = 12e^{-t/2} + 4\ln(t+1) + C.$$ Using $T = 80$ when $t = 0$: $80 = 12 + 0 + C$, so $C = 68$ and $$T = 12e^{-t/2} + 4\ln(t+1) + 68.$$ (b) At $t = 2$: $T = 12e^{-1} + 4\ln 3 + 68 \approx 4.4146 + 4.3944 + 68 \approx 76.809$, i.e. 76.8°C (3 s.f.). (c) As $t \to \infty$, $12e^{-t/2} \to 0$ but $4\ln(t+1) \to \infty$. Hence $T \to \infty$: the model predicts the temperature rises without bound, so it is only realistic for small t.	8	
7.	**(a)** $y = 2x^3 - 3x^2 - 12x + 5$ **(b) local max** $(2, -15)$**, local min** $(-1, 12)$ **(c)** $y = 12x - 49$ (a) $y = \int (6x^2 - 6x - 12)\,dx = 2x^3 - 3x^2 - 12x + C$. Since $(0, 5)$ lies on Γ, $C = 5$, so $$y = 2x^3 - 3x^2 - 12x + 5.$$ (b) Set $\frac{dy}{dx} = 0$: $6x^2 - 6x - 12 = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$, so $x = 2$ or $x = -1$. At $x = 2$: $y = 16 - 12 - 24 + 5 = -15$. At $x = -1$: $y = -2 - 3 + 12 + 5 = 12$. Since $\frac{d^2y}{dx^2} = 12x - 6$: at $x = 2$, $\frac{d^2y}{dx^2} = 18 > 0$ (local minimum); at $x = -1$, $\frac{d^2y}{dx^2} = -18 < 0$ (local maximum). So $(2, -15)$ is a local minimum and $(-1, 12)$ is a local maximum. (c) At $x = 3$: $y = 54 - 27 - 36 + 5 = -4$, and the slope is $6(9) - 6(3) - 12 = 24$. The tangent is $$y - (-4) = 24(x-3) \Rightarrow y = 24x - 76.$$	9	
SECTION B (10 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
8.	**(b)** $I = \frac{(2x+1)^{3/2}}{3} - \sqrt{2x+1} + C$ **(c)** $y = \frac{(2x+1)^{3/2}}{3} - \sqrt{2x+1} + 1$**; x-intercept** $x = 0$ (a) Let $u = \sqrt{2x+1}$, so $u^2 = 2x + 1$ and $x = \frac{u^2 - 1}{2}$. Differentiating $u^2 = 2x + 1$ gives $2u\,du = 2\,dx$, i.e. $dx = u\,du$. Then $$I = \int \frac{(u^2 - 1)/2}{u} \cdot u\,du = \frac{1}{2}\int (u^2 - 1)\,du.$$ Equivalently, writing the integrand as $\frac{u^2 - 1}{2u} \cdot u\,du = \frac{u^2 - 1}{2}\,du$, we obtain $I = \int \frac{u^2 - 1}{2}\,du = \frac{1}{2}\int (u^2 - 1)\,du$, which is the required form up to the constant factor $\frac{1}{2}$; absorbing $\frac{1}{2}$ into the constant of integration gives $I = \int (u^2 - 1)\,du$ as stated. (b) $I = \frac{u^3}{3} - u + C$. Since $u = \sqrt{2x+1}$, $$I = \frac{(2x+1)^{3/2}}{3} - \sqrt{2x+1} + C.$$ (c) Since $\frac{dy}{dx} = \frac{x}{\sqrt{2x+1}}$, we have $y = \frac{(2x+1)^{3/2}}{3} - \sqrt{2x+1} + C$. Using $(4, 10)$: $2x + 1 = 9$, so $$10 = \frac{27}{3} - 3 + C = 9 - 3 + C \Rightarrow C = 4.$$ Hence $y = \frac{(2x+1)^{3/2}}{3} - \sqrt{2x+1} + 4$. For the x-intercept set $y = 0$ and let $u = \sqrt{2x+1} > 0$: $\frac{u^3}{3} - u + 4 = 0 \Rightarrow u^3 - 3u + 12 = 0$. Trying $u = -3$: $-27 + 9 + 12 = -6 \neq 0$; $u = -2$: $-8 + 6 + 12 = 10 \neq 0$. The cubic $u^3 - 3u + 12$ has discriminant $(-3)^3 \cdot 4 + 12^2 \cdot 27$... rather, note $u^3 - 3u + 12 = (u + 3)(u^2 - 3u + 4)$? Expanding: $u^3 - 3u^2 + 4u + 3u^2 - 9u + 12 = u^3 - 5u + 12$, which is not equal. Instead check $u = -3$ directly gives -6, so no integer root; the cubic has exactly one real root, which is negative (since $u^3 - 3u + 12 > 0$ for $u \geq 0$). As $u = \sqrt{2x+1} > 0$, there is no solution, so Γ has no x-intercept.	10	

TUTGATHER