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TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

# **MATHEMATICS** Extended Part Module M1
# PAPER

## Question-Answer Book

Candidate Number

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

*Topic practice: Applications of Differentiation*

### INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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**SECTION A(1) (16 marks)**

1. Consider the curve $C$: $y = x^3 - 4x + 1$. **(3 marks)**

(a) Find $\frac{dy}{dx}$.

(b) Find the equation of the tangent to $C$ at the point where $x = 2$, giving your answer in the form $y = mx + c$.

2. Let $f(x) = x^3 - 6x^2 + 9x + 2$. **(3 marks)**

(a) Find $f'(x)$.

(b) Solve $f'(x) = 0$ and hence find the coordinates of the stationary points of the graph of $y = f(x)$.

3. Let $f(x) = x^3 - 3x^2 - 9x + 5$ for $-2 \leq x \leq 5$. **(4 marks)**

(a) Find $f'(x)$ and the values of $x$ in $-2 < x < 5$ where $f'(x) = 0$.

(b) Hence find the global maximum value and the global minimum value of $f(x)$ on the interval $-2 \leq x \leq 5$.

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4. A rectangular enclosure is to be built against a straight wall using 40 m of fencing. The wall forms one side of the enclosure, and fencing is used for the other three sides. Let $x$ m be the length of each of the two sides perpendicular to the wall, and let $y$ m be the length of the side parallel to the wall. **(6 marks)**

(a) Show that the area $A$ m$^2$ of the enclosure is given by $A = 40x - 2x^2$.

(b) Find the value of $x$ that maximises $A$, and find the maximum area.

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**SECTION A(2) (24 marks)**

5. Consider the curve $C$: $y = \frac{x^2 + 3}{x + 1}$, where $x \neq -1$. **(7 marks)**

(a) Find $\frac{dy}{dx}$.

(b) Show that the tangent to $C$ at the point where $x = 1$ has equation $y = x + 2$.

(c) Find the equation of the normal to $C$ at the point where $x = 1$, giving your answer in the form $x + y = k$.

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6. Let $f(x) = 2x^3 - 9x^2 + 12x - 4$. **(8 marks)**

(a) Find $\dot{f}(x)$ and $\ddot{f}(x)$.

(b) Find the coordinates of the stationary points of the graph of $y = f(x)$ and determine the nature of each using the second derivative test.

(c) Find the $x$-coordinates of the points of inflexion of the graph of $y = f(x)$.

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7. A closed cylindrical can has radius $r$ cm and height $h$ cm. Its total surface area is fixed at $150\pi$ cm$^2$. **(9 marks)**

(a) Show that the volume $V$ cm$^3$ of the can is given by $V = 75\pi r - \pi r^3$.

(b) Find the value of $r$ that maximises $V$, and justify that this value gives the maximum volume.

(c) Hence find the maximum volume of the can, and the corresponding value of $h$.

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**SECTION B (10 marks)**

8. Consider the curve $C$: $y = x^3 - 3x^2 + 4$. **(10 marks)**

(a) Find the coordinates of the stationary points of $C$ and determine their nature.

(b) Find the equation of the tangent $T$ to $C$ at the point $P(3, 4)$.

(c) Show that $T$ meets $C$ again at a point $Q$, and find the coordinates of $Q$.

(d) Find the area of the region enclosed between $C$ and $T$.

(e) A point moves along $C$ so that its $x$-coordinate increases at a constant rate of 0.5 units per second. Find the rate of change of the $y$-coordinate at the instant when $x = 2$.

**END OF PAPER**

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