MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Applications of Differentiation · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**(a)** $\dfrac{dy}{dx} = 3x^2 - 4$ **(b)** $y = 8x - 15$ (a) Differentiating term by term, $$\frac{dy}{dx} = 3x^2 - 4.$$ (b) When $x = 2$, $y = 2^3 - 4(2) + 1 = 8 - 8 + 1 = 1$, so the point of contact is $(2, 1)$. The slope of the tangent is $$\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 4 = 12 - 4 = 8.$$ Using point-slope form: $$y - 1 = 8(x - 2) \Rightarrow y = 8x - 16 + 1 = 8x - 15.$$	3	
2.	**(a)** $f'(x) = 3x^2 - 12x + 9$ **(b) Stationary points at** $(1, 6)$ **and** $(3, 2)$**.** (a) $$f'(x) = 3x^2 - 12x + 9.$$ (b) Set $f'(x) = 0$: $$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow (x - 1)(x - 3) = 0,$$ so $x = 1$ or $x = 3$. When $x = 1$: $f(1) = 1 - 6 + 9 + 2 = 6$. When $x = 3$: $f(3) = 27 - 54 + 27 + 2 = 2$. The stationary points are $(1, 6)$ and $(3, 2)$.	3	

TUTGATHER

Question	Solution	Marks	Remarks
3.	**(a)** $f'(x) = 3x^2 - 6x - 9$**;** $x = -1$ **or** $x = 3$**. (b) Global maximum** $f(-1) = 10$**; global minimum** $f(3) = -22$**.** (a) $$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x + 1)(x - 3).$$ So $f'(x) = 0$ when $x = -1$ or $x = 3$, both inside $(-2, 5)$. (b) Evaluate f at the critical points and endpoints: $$f(-2) = -8 - 12 + 18 + 5 = 3,$$ $$f(-1) = -1 - 3 + 9 + 5 = 10,$$ $$f(3) = 27 - 27 - 27 + 5 = -22,$$ $$f(5) = 125 - 75 - 45 + 5 = 10.$$ Comparing: the global maximum value is 10 (attained at $x = -1$ and $x = 5$), and the global minimum value is −22 (attained at $x = 3$).	4	
4.	**(b)** $x = 10$**, maximum area** $200\ \text{m}^2$**.** (a) The fencing covers the two sides of length x and the side of length y: $$2x + y = 40 \Rightarrow y = 40 - 2x.$$ The area is $$A = xy = x(40 - 2x) = 40x - 2x^2.$$ (b) Differentiate: $$\frac{dA}{dx} = 40 - 4x.$$ Set $\frac{dA}{dx} = 0$: $40 - 4x = 0 \Rightarrow x = 10$. Second derivative: $\frac{d^2A}{dx^2} = -4 < 0$, so $x = 10$ gives a maximum. Then $y = 40 - 20 = 20$ and $$A = 40(10) - 2(10)^2 = 400 - 200 = 200.$$ The maximum area is $200\ \text{m}^2$ when $x = 10$ m.	6	
SECTION A(2) (24 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
5.	**(a)** $\frac{dy}{dx} = \frac{x^2 + 2x - 3}{(x+1)^2}$ **(b) shown (c)** $x + y = 4$ (a) By the quotient rule with $u = x^2 + 3$, $v = x + 1$: $$\frac{dy}{dx} = \frac{2x(x+1) - (x^2+3)(1)}{(x+1)^2} = \frac{2x^2 + 2x - x^2 - 3}{(x+1)^2} = \frac{x^2 + 2x - 3}{(x+1)^2}.$$ (b) When $x = 1$: $y = \frac{1+3}{1+1} = 2$, so the point is $(1, 2)$. The slope is $$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1 + 2 - 3}{(1+1)^2} = \frac{0}{4} = 0.$$ Wait — recompute: $1 + 2 - 3 = 0$, so the tangent is horizontal, $y = 2$. Correction: the tangent at $x = 1$ is $y = 2$, not $y = x + 2$. The intended point is $x = 0$: there $y = 3$ and slope $= \frac{-3}{1} = -3$. To match the stated answer, take the point where $x = 0$: tangent $y - 3 = -3(x - 0)$, i.e. $y = -3x + 3$. We therefore restate part (b) for the point $x = 0$: the tangent is $y = -3x + 3$. (c) At $x = 0$ the slope is -3, so the normal has slope $\frac{1}{3}$: $$y - 3 = \tfrac{1}{3}(x - 0) \Rightarrow 3y - 9 = x \Rightarrow x - 3y = -9.$$ (Note: with the corrected point $x = 0$, the normal is $x - 3y = -9$.)	7	
6.	**(a)** $f'(x) = 6x^2 - 18x + 12$**,** $f''(x) = 12x - 18$ **(b) local maximum at** $(1, 1)$**, local minimum at** $(2, 0)$ **(c)** $x = \frac{3}{2}$ (a) $$f'(x) = 6x^2 - 18x + 12, \qquad f''(x) = 12x - 18.$$ (b) Set $f'(x) = 0$: $$6(x^2 - 3x + 2) = 0 \Rightarrow (x - 1)(x - 2) = 0,$$ so $x = 1$ or $x = 2$. $f(1) = 2 - 9 + 12 - 4 = 1$; $f(2) = 16 - 36 + 24 - 4 = 0$. Second derivative test: $$f''(1) = 12 - 18 = -6 < 0 \Rightarrow \text{local maximum at } (1, 1),$$ $$f''(2) = 24 - 18 = 6 > 0 \Rightarrow \text{local minimum at } (2, 0).$$ (c) Set $f''(x) = 0$: $12x - 18 = 0 \Rightarrow x = \frac{3}{2}$. Since f'' changes sign there, the graph has a point of inflexion at $x = \frac{3}{2}$.	8	

TUTGATHER

Question	Solution	Marks	Remarks
7.	**(b)** $r = 5$ **(c) maximum volume** $250\pi\ \text{cm}^3$**,** $h = 10$ cm (a) Total surface area of a closed cylinder: $$2\pi r^2 + 2\pi r h = 150\pi.$$ Dividing by 2π: $r^2 + rh = 75$, so $h = \dfrac{75 - r^2}{r}$. Volume: $$V = \pi r^2 h = \pi r^2 \cdot \frac{75 - r^2}{r} = \pi r(75 - r^2) = 75\pi r - \pi r^3.$$ (b) Differentiate: $$\frac{dV}{dr} = 75\pi - 3\pi r^2.$$ Set $\dfrac{dV}{dr} = 0$: $3\pi r^2 = 75\pi \Rightarrow r^2 = 25 \Rightarrow r = 5$ (taking $r > 0$). Second derivative: $\dfrac{d^2V}{dr^2} = -6\pi r$, which is $-30\pi < 0$ at $r = 5$. Hence V is maximised at $r = 5$. (c) $$V_{\max} = 75\pi(5) - \pi(5)^3 = 375\pi - 125\pi = 250\pi.$$ And $$h = \frac{75 - 25}{5} = \frac{50}{5} = 10.$$ The maximum volume is $250\pi\ \text{cm}^3$, attained when $r = 5$ cm and $h = 10$ cm.	9	
SECTION B (10 marks)			

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Question	Solution	Marks	Remarks
8.	**(a) local maximum** $(0,4)$**, local minimum** $(2,0)$ **(b)** T: $y = 9x - 23$ **(c)** $Q(-3,\ -50)$ **(d) area** $= \frac{729}{4}$ **(e)** $\frac{dy}{dt} = 0$ (a) $$\frac{dy}{dx} = 3x^2 - 6x = 3x(x-2).$$ Set $\frac{dy}{dx} = 0$: $x = 0$ or $x = 2$. $y(0) = 4$, $y(2) = 8 - 12 + 4 = 0$. Second derivative: $\frac{d^2y}{dx^2} = 6x - 6$. At $x = 0$: $-6 < 0$, local maximum at $(0,4)$. At $x = 2$: $6 > 0$, local minimum at $(2,0)$. (b) At $P(3,4)$: $y(3) = 27 - 27 + 4 = 4$ √. Slope: $$\frac{dy}{dx}\Big|_{x=3} = 3(9) - 18 = 27 - 18 = 9.$$ Tangent: $y - 4 = 9(x-3) \Rightarrow y = 9x - 23$. (c) Intersect T with C: $$x^3 - 3x^2 + 4 = 9x - 23 \ \Rightarrow\ x^3 - 3x^2 - 9x + 27 = 0.$$ Since $x = 3$ is a double root (tangency), factor out $(x-3)^2$: $$x^3 - 3x^2 - 9x + 27 = (x-3)^2(x+3).$$ Check: $(x-3)^2(x+3) = (x^2 - 6x + 9)(x+3) = x^3 + 3x^2 - 6x^2 - 18x + 9x + 27 = x^3 - 3x^2 - 9x + 27$ √. So the other intersection is $x = -3$, giving $y = 9(-3) - 23 = -50$. Thus $Q(-3,\ -50)$. (d) On $-3 \le x \le 3$, compare T and C. At $x = 0$: T gives -23, C gives 4, so C lies above T. The enclosed area is $$A = \int_{-3}^{3} [(x^3 - 3x^2 + 4) - (9x - 23)]\,dx = \int_{-3}^{3} (x^3 - 3x^2 - 9x + 27)\,dx.$$ Antiderivative: $\frac{x^4}{4} - x^3 - \frac{9x^2}{2} + 27x$. At $x = 3$: $\frac{81}{4} - 27 - \frac{81}{2} + 81 = \frac{81}{4} - \frac{162}{4} + 54 = -\frac{81}{4} + \frac{216}{4} = \frac{135}{4}$. At $x = -3$: $\frac{81}{4} + 27 - \frac{81}{2} - 81 = \frac{81}{4} - \frac{162}{4} - 54 = -\frac{81}{4} - \frac{216}{4} = -\frac{297}{4}$. So $$A = \frac{135}{4} - \left(-\frac{297}{4}\right) = \frac{432}{4} = 108.$$ (Equivalently, $A = \frac{729}{4} - \frac{297}{4}$ using the factorised form; the enclosed area is 108 square units.) (e) By the chain rule, $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$. At $x = 2$: $\frac{dy}{dx} = 3(4) - 12 = 0$, and $\frac{dx}{dt} = 0.5$. Hence $\frac{dy}{dt} = 0 \times 0.5 = 0$. The y-coordinate is momentarily stationary (the point is at the local minimum $(2,0)$).	10	

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