MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Derivative of a Function · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**6** Factorise the numerator: $x^2 - 9 = (x-3)(x+3)$. For $x \neq 3$, $\frac{x^2-9}{x-3} = x+3$. Hence $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} (x+3) = 6$.	3	
2.	**2** Let $u = 2x$. As $x \to 0$, $u \to 0$. Then $\frac{e^{2x}-1}{x} = 2 \cdot \frac{e^u - 1}{u}$. Using the standard limit $\lim_{u \to 0} \frac{e^u - 1}{u} = 1$, the limit is $2 \times 1 = 2$.	3	
3.	$2x + 3$ By definition, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Now $f(x+h) = (x+h)^2 + 3(x+h) = x^2 + 2xh + h^2 + 3x + 3h$. Thus $f(x+h) - f(x) = 2xh + h^2 + 3h = h(2x + h + 3)$. For $h \neq 0$, $\frac{f(x+h) - f(x)}{h} = 2x + h + 3$. Taking $h \to 0$ gives $f'(x) = 2x + 3$.	4	
4.	**(a)** $\lim_{x \to 2^-} f(x) = 5$**,** $\lim_{x \to 2^+} f(x) = 5$**. (b) The limit exists and equals** 5**.** (a) For $x < 2$, $f(x) = x^2 + 1$, so $\lim_{x \to 2^-} f(x) = 2^2 + 1 = 5$. For $x > 2$, $f(x) = 3x - 1$, so $\lim_{x \to 2^+} f(x) = 3(2) - 1 = 5$. (b) Since the left-hand and right-hand limits are equal, $\lim_{x \to 2} f(x)$ exists and equals 5.	6	
SECTION A(2) (24 marks)			
5.	**(a)** 5**; (b)** 3 (a) Let $u = 5x$. As $x \to 0$, $u \to 0$. Then $\frac{\ln(1+5x)}{x} = 5 \cdot \frac{\ln(1+u)}{u}$. Using $\lim_{u \to 0} \frac{\ln(1+u)}{u} = 1$, the limit is $5 \times 1 = 5$. (b) Divide numerator and denominator by e^{2x}: $\frac{3 + 2e^{-3x}}{1 - 5e^{-2x}}$. As $x \to \infty$, $e^{-3x} \to 0$ and $e^{-2x} \to 0$. Hence the limit is $\frac{3+0}{1-0} = 3$.	7	

TUTGATHER

Question	Solution	Marks	Remarks
6.	**(a)** $f'(x) = \frac{1}{\sqrt{2x+1}}$**; (b)** $y = \frac{1}{3}x + \frac{5}{3}$ (a) $f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$. Rationalise the numerator: multiply by $\frac{\sqrt{2x+2h+1}+\sqrt{2x+1}}{\sqrt{2x+2h+1}+\sqrt{2x+1}}$. The numerator becomes $(2x+2h+1)-(2x+1) = 2h$. Thus the difference quotient is $\frac{2h}{h(\sqrt{2x+2h+1}+\sqrt{2x+1})} = \frac{2}{\sqrt{2x+2h+1}+\sqrt{2x+1}}$. Taking $h \to 0$ gives $f'(x) = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$. (b) At $x = 4$, $f(4) = \sqrt{9} = 3$ and $f'(4) = \frac{1}{3}$. The tangent has equation $y - 3 = \frac{1}{3}(x-4)$, i.e. $y = \frac{1}{3}x + \frac{5}{3}$.	8	
7.	**(a)** $x + 2$**; (b)** 4**; (c) Yes,** $f(2) = 4$**.** (a) $\frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x + 2$ for $x \neq 2$. (b) $\lim_{x \to 2} f(x) = \lim_{x \to 2}(x+2) = 4$. (c) Since the limit exists and equals 4, defining $f(2) = 4$ makes f continuous at $x = 2$.	9	
SECTION B (10 marks)			
8.	**(a)** $f(x) = e^{2x} + e^{x} + 1$**; (b)** 3**; (c)** 3 (a) For $x \neq 0$, $e^x - 1 \neq 0$, so $f(x) = \frac{(e^x-1)(e^{2x}+e^x+1)}{e^x-1} = e^{2x} + e^x + 1$. (b) $\lim_{x \to 0} f(x) = e^0 + e^0 + 1 = 1 + 1 + 1 = 3$. (c) $\frac{f(x)-3}{x} = \frac{e^{2x}+e^x+1-3}{x} = \frac{e^{2x}-1}{x} + \frac{e^x-1}{x}$. Using $\lim_{x \to 0} \frac{e^{kx}-1}{x} = k$, we get $\lim_{x \to 0} \frac{e^{2x}-1}{x} = 2$ and $\lim_{x \to 0} \frac{e^x-1}{x} = 1$. Hence the limit is $2 + 1 = 3$.	10	

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