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**TUTGATHER PRACTICE PAPER**

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

# **MATHEMATICS** Extended Part Module M1
# PAPER

## Question-Answer Book

Candidate Number

**TUTGATHER**
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

*Topic practice: Exponential and Logarithmic Functions*

### INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

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**SECTION A(1) (16 marks)**

1. Simplify $\frac{e^{2x} - e^{-2x}}{e^{x} + e^{-x}}$. **(3 marks)**

2. Expand $e^{3x}$ in ascending powers of $x$ up to and including the term in $x^3$. **(3 marks)**

3. The population $P$ of a bacteria culture after $t$ hours is modelled by $P = P_0 e^{kt}$, where $P_0$ and $k$ are positive constants. Initially there are 500 bacteria, and after 3 hours the population is 4000. Find the value of $k$ correct to 3 significant figures. **(4 marks)**

Answers written in the margins will not be marked.
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4. The variables $x$ and $y$ satisfy $y = ae^{bx}$, where $a$ and $b$ are constants. Taking natural logarithms, we obtain $\ln y = \ln a + bx$. The following data are obtained: **(6 marks)**

| $x$ | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| $y$ | 5.0 | 9.0 | 16.2 | 29.2 |

(a) Complete the table of values of $\ln y$, correct to 3 decimal places.

(b) Plot $\ln y$ against $x$ on a graph paper and draw a straight line of best fit.

(c) Use your graph to estimate the values of $a$ and $b$.

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**SECTION A(2) (24 marks)**

5. The function $f$ is defined by $f(x) = \frac{e^x - 1}{x}$ for $x \neq 0$. **(7 marks)**

(a) Expand $e^x$ in ascending powers of $x$ up to the term in $x^3$.

(b) Hence show that $f(x) \approx 1 + \frac{x}{2} + \frac{x^2}{6}$ for small values of $x$.

(c) Find $\lim_{x \to 0} f(x)$.

6. (a) Expand $e^{2x}$ in ascending powers of $x$ up to the term in $x^3$. **(8 marks)**

(b) Find the coefficient of $x^3$ in the expansion of $(1 - x)^5 e^{2x}$.

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7. The temperature $T$ (in $^\circ$C) of a cup of coffee after $t$ minutes is modelled by $T = 20 + 60e^{-kt}$, where $k$ is a positive constant. Initially the temperature is 80$^\circ$C, and after 5 minutes it is 50$^\circ$C. **(9 marks)**

(a) Find the value of $k$ correct to 3 significant figures.

(b) Find the temperature after 10 minutes, correct to the nearest $^\circ$C.

(c) How long will it take for the temperature to drop to 30$^\circ$C? Give the answer correct to the nearest minute.

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**SECTION B (10 marks)**

8. In an experiment, the variable $y$ is believed to be related to $x$ by the equation $y = Ae^{kx}$, where $A$ and $k$ are constants. Taking natural logarithms gives $\ln y = \ln A + kx$. The following data are recorded: **(10 marks)**

| $x$ | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| $y$ | 2.5 | 4.1 | 6.7 | 11.0 | 18.0 |

(a) Complete the table of values of $\ln y$, correct to 3 decimal places.

(b) Plot $\ln y$ against $x$ and draw a straight line of best fit.

(c) Use your graph to estimate $A$ and $k$.

(d) Using the model, predict the value of $y$ when $x = 6$, correct to 3 significant figures.

**END OF PAPER**

Answers written in the margins will not be marked.

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