MARKING SCHEME

MATHEMATICS Extended Part Module M1 · Paper

Topic practice: Exponential and Logarithmic Functions · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$e^{x} - e^{-x}$ Factor the numerator as a difference of squares: $e^{2x} - e^{-2x} = (e^{x} - e^{-x})(e^{x} + e^{-x})$. Then cancel the common factor $e^{x} + e^{-x}$ (which is never zero) to obtain $e^{x} - e^{-x}$.	3	
2.	$1 + 3x + \frac{9}{2}x^{2} + \frac{9}{2}x^{3} + \cdots$ Using the exponential series $e^{u} = 1 + u + \frac{u^{2}}{2!} + \frac{u^{3}}{3!} + \cdots$ with $u = 3x$: $e^{3x} = 1 + 3x + \frac{(3x)^{2}}{2} + \frac{(3x)^{3}}{6} + \cdots = 1 + 3x + \frac{9}{2}x^{2} + \frac{27}{6}x^{3} + \cdots = 1 + 3x + \frac{9}{2}x^{2} + \frac{9}{2}x^{3} + \cdots$.	3	
3.	$k \approx 0.693$ At $t = 0$, $P = 500$, so $P_{0} = 500$. At $t = 3$, $P = 4000$: $500e^{3k} = 4000 \Rightarrow e^{3k} = 8 \Rightarrow 3k = \ln 8 \Rightarrow k = \frac{\ln 8}{3} \approx 0.693$ (to 3 s.f.).	4	
4.	**(a)** $\ln y$**: 1.609, 2.197, 2.785, 3.374. (c)** $a \approx 2.8$**,** $b \approx 0.59$**.** (a) $\ln 5.0 \approx 1.609$, $\ln 9.0 \approx 2.197$, $\ln 16.2 \approx 2.785$, $\ln 29.2 \approx 3.374$. (b) Plot the points $(1, 1.609)$, $(2, 2.197)$, $(3, 2.785)$, $(4, 3.374)$ and draw a straight line. (c) The line has equation $\ln y = \ln a + bx$. The y-intercept is $\ln a$. From the graph, the line crosses the vertical axis at approximately 1.03, so $\ln a \approx 1.03$ and $a \approx e^{1.03} \approx 2.8$. The gradient is $b \approx \frac{3.374 - 1.609}{4 - 1} = \frac{1.765}{3} \approx 0.588$, so $b \approx 0.59$.	6	
SECTION A(2) (24 marks)			
5.	**(a)** $e^{x} = 1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + \cdots$ **(b) shown (c)** 1 (a) $e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \cdots = 1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + \cdots$. (b) $e^{x} - 1 = x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + \cdots$. Dividing by x (for $x \neq 0$): $f(x) = 1 + \frac{x}{2} + \frac{x^{2}}{6} + \cdots$. For small x, the higher powers are negligible, so $f(x) \approx 1 + \frac{x}{2} + \frac{x^{2}}{6}$. (c) As $x \to 0$, $f(x) \to 1 + 0 + 0 = 1$. Hence $\lim_{x \to 0} f(x) = 1$.	7	

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Question	Solution	Marks	Remarks
6.	**(a)** $1 + 2x + 2x^2 + \frac{4}{3}x^3 + \cdots$ **(b)** $-\frac{26}{3}$ (a) $e^{2x} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \cdots = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \cdots$. (b) $(1-x)^5 = 1 - 5x + 10x^2 - 10x^3 + \cdots$. Multiplying the two series up to x^3: $(1-x)^5 e^{2x} = (1 - 5x + 10x^2 - 10x^3)(1 + 2x + 2x^2 + \frac{4}{3}x^3) + \cdots$ $= 1 + (2-5)x + (2 - 10 + 10)x^2 + (\frac{4}{3} - 10 + 20 - 10)x^3 + \cdots$ $= 1 - 3x + 2x^2 - \frac{26}{3}x^3 + \cdots$ Thus the coefficient of x^3 is $-\frac{26}{3}$.	8	
7.	**(a)** $k \approx 0.135$ **(b)** 35°C **(c) 18 minutes** (a) At $t = 0$, $T = 80$: $20 + 60e^0 = 80$, which is consistent. At $t = 5$, $T = 50$: $20 + 60e^{-5k} = 50 \Rightarrow 60e^{-5k} = 30 \Rightarrow e^{-5k} = 0.5 \Rightarrow -5k = \ln 0.5 \Rightarrow k = \frac{\ln 2}{5} \approx 0.1386 \approx 0.139$ (to 3 s.f.). (b) $T(10) = 20 + 60e^{-10k} = 20 + 60(e^{-5k})^2 = 20 + 60(0.5)^2 = 20 + 15 = 35^\circ$C. (c) Set $T = 30$: $20 + 60e^{-kt} = 30 \Rightarrow 60e^{-kt} = 10 \Rightarrow e^{-kt} = \frac{1}{6} \Rightarrow -kt = \ln\frac{1}{6} = -\ln 6 \Rightarrow t = \frac{\ln 6}{k} = \frac{5\ln 6}{\ln 2} \approx 12.9$ minutes. To the nearest minute, $t \approx 13$ minutes.	9	
SECTION B (10 marks)			
8.	**(a)** $\ln y$**: 0.916, 1.411, 1.902, 2.398, 2.890. (c)** $A \approx 1.5$**,** $k \approx 0.49$**. (d)** $y \approx 29.5$**.** (a) $\ln 2.5 \approx 0.916$, $\ln 4.1 \approx 1.411$, $\ln 6.7 \approx 1.902$, $\ln 11.0 \approx 2.398$, $\ln 18.0 \approx 2.890$. (b) Plot the points $(1, 0.916)$, $(2, 1.411)$, $(3, 1.902)$, $(4, 2.398)$, $(5, 2.890)$ and draw a straight line. (c) The line has equation $\ln y = \ln A + kx$. The y-intercept is $\ln A$. From the graph, the line crosses the vertical axis at approximately 0.42, so $\ln A \approx 0.42$ and $A \approx e^{0.42} \approx 1.5$. The gradient is $k \approx \frac{2.890 - 0.916}{5 - 1} = \frac{1.974}{4} \approx 0.4935$, so $k \approx 0.49$. (d) When $x = 6$, $\ln y \approx 0.42 + 0.4935 \times 6 = 0.42 + 2.961 = 3.381$. Thus $y \approx e^{3.381} \approx 29.4$ (to 3 s.f.).	10	

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