Please stick the barcode label here.

TUTGATHER PRACTICE PAPER

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2027

Candidate Number

MATHEMATICS Compulsory Part
PAPER 1

Question-Answer Book

TUTGATHER
PRACTICE PAPER · NOT AN OFFICIAL HKEAA PAPER

1 hour 5 minutes

This paper must be answered in English

Topic practice: Inquiry and Investigation

INSTRUCTIONS

(1) This paper consists of THREE sections, A(1), A(2) and B.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(5) The diagrams in this paper are not necessarily drawn to scale.

(6) Calculators and mathematical drawing instruments may be used.

(7) The marks for each question are shown in brackets.

(8) No extra time will be given after the 'Time is up' announcement.

TUTGATHER

SECTION A(1) (16 marks)

1. A sequence is defined by $a_1 = 2$ and $a_{n+1} = 3a_n - 4$ for $n \geq 1$. Write down the first four terms of the sequence and describe the pattern of the sequence. **(3 marks)**

2. The cost C (in dollars) of producing n items is modelled by $C = 500 + 12n$. (a) Find the cost of producing 20 items. (b) If the budget is \$2000, find the maximum number of items that can be produced. **(3 marks)**

3. A rectangular garden is to be fenced on three sides (the fourth side is a wall). The total length of fencing available is 40 m. Let the length of the side parallel to the wall be x m. (a) Express the area A of the garden in terms of x. (b) Find the maximum possible area. **(4 marks)**

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

TUTGATHER

4. A student claims: “For any positive integer n, $n^2 + n + 41$ is a prime number.” (a) Test the claim for $n = 1, 2, 3$. (b) Show that the claim is false by finding a counterexample. (c) Explain why testing several values is not sufficient to prove a general statement. **(6 marks)**

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

TUTGATHER

SECTION A(2) (24 marks)

5. A sequence is defined by $u_1 = 1$ and $u_{n+1} = \frac{u_n}{2u_n + 1}$ for $n \geq 1$. (a) Find u_2 and u_3. (b) Make a conjecture for a formula for u_n in terms of n. (c) Prove your conjecture by mathematical induction. **(7 marks)**

6. The population P (in thousands) of a bacteria culture after t hours is modelled by $P = P_0 e^{kt}$. Initially $P = 2$ (thousand). After 3 hours, $P = 16$ (thousand). (a) Find P_0 and k (correct to 3 significant figures). (b) Find the population after 5 hours. (c) Find the time required for the population to reach 100 thousand. (d) State one limitation of this model. **(8 marks)**

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

TUTGATHER

7. A cylindrical water tank has radius r m and height h m. Its volume is 100 m^3. The material for the curved surface costs \$20 per m^2, and for the top and bottom costs \$30 per m^2. (a) Express h in terms of r. (b) Show that the total cost C (in dollars) is $C = 60\pi r^2 + \frac{4000}{r}$. (c) Find the value of r that minimises the cost, and the minimum cost (correct to 3 significant figures). **(9 marks)**

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

TUTGATHER

SECTION B (10 marks)

8. A sequence is defined by $x_1 = 1$ and $x_{n+1} = \frac{x_n + 2}{x_n + 1}$ for $n \geq 1$. (a) Find x_2 and x_3. (b) Prove by mathematical induction that $x_n > 0$ for all positive integers n. (c) Show that $x_{n+1} - \sqrt{2} = \frac{(x_n - \sqrt{2})(1 - \sqrt{2})}{x_n + 1}$. (d) Hence prove that $x_n > \sqrt{2}$ for all $n \geq 2$. (e) Determine whether the sequence is increasing or decreasing for $n \geq 2$, and justify your answer. **(10 marks)**

END OF PAPER

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

TUTGATHER