MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: Further Applications · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	\$22 538 **(to the nearest dollar)** Quarterly interest rate $= \frac{4\%}{4} = 1\%$ per quarter; number of quarters $= 3 \times 4 = 12$. $A = 20\,000(1 + \frac{0.04}{4})^{12} = 20\,000(1.01)^{12}$. $(1.01)^{12} \approx 1.126825$, so $A \approx 20\,000 \times 1.126825 = 22\,536.5$. Rounding to the nearest dollar gives \$22 537. (Accept \$22 536–\$22 538 depending on rounding of the intermediate value; the exact value is $20\,000(1.01)^{12} =$ \$22 536.50 … , i.e. \$22 537.)	3	
2.	**(a)** $\hat{p} = 0.64$ **(b) standard error** ≈ 0.0304 (a) $\hat{p} = \frac{160}{250} = 0.64$. (b) Standard error $= \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.64 \times 0.36}{250}} = \sqrt{\frac{0.2304}{250}} = \sqrt{0.0009216} \approx 0.0304$ (3 s.f.).	3	
3.	$\angle BCD = 70^\circ$ **(exterior angle of cyclic quadrilateral);** $\angle ADC = 75^\circ$ Since $ABCD$ is cyclic, the exterior angle at B equals the interior opposite angle at D: $\angle BCD = \angle CBE = 70^\circ$ (exterior angle of a cyclic quadrilateral equals the interior opposite angle). Opposite angles of a cyclic quadrilateral are supplementary, so $\angle ADC = 180^\circ - \angle ABC$. Now $\angle ABC = 180^\circ - \angle CBE = 180^\circ - 70^\circ = 110^\circ$ (adjacent angles on the straight line ABE). Hence $\angle ADC = 180^\circ - 110^\circ = 70^\circ$. Check with the other pair: $\angle BAD + \angle BCD = 105^\circ + 70^\circ = 175^\circ \neq 180^\circ$, so the data are inconsistent; using the given $\angle BAD = 105^\circ$ together with the cyclic property gives $\angle BCD = 180^\circ - 105^\circ = 75^\circ$ and then $\angle ADC = 180^\circ - \angle ABC = 70^\circ$. Taking the consistent reading: $\angle BCD = 75^\circ$ and $\angle ADC = 70^\circ$.	4	
4.	**(a) length** $= (40 - 2x)$ **cm, width** $= (30 - 2x)$ **cm (b) shown (c)** $0 < x < 15$ (a) Cutting a square of side x from each corner reduces each dimension by $2x$: length $= (40 - 2x)$ cm, width $= (30 - 2x)$ cm. (b) $V = x(40 - 2x)(30 - 2x) = x(1200 - 80x - 60x + 4x^2) = x(4x^2 - 140x + 1200) = 4x^3 - 140x^2 + 1200x$. (c) Both dimensions must be positive: $40 - 2x > 0$ and $30 - 2x > 0$, so $x < 15$; also $x > 0$. Hence $0 < x < 15$.	6	
SECTION A(2) (24 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
5.	**(a)** 0.3% **per month (b) shown (c)** $R \approx \$2\,983$ (a) Monthly rate $i = \frac{3.6\%}{12} = 0.3\% = 0.003$. (b) Let A_n be the amount owed after n months. Each month the debt is multiplied by 1.003 and then reduced by R: $$A_n = 1.003A_{n-1} - R, \quad A_0 = 300\,000.$$ Solving the recurrence, $A_n = 300\,000(1.003)^n - R(1 + 1.003 + \cdots + 1.003^{n-1}) = 300\,000(1.003)^n - R \cdot \frac{(1.003)^n - 1}{0.003}$. (c) The loan is cleared after $n = 120$ months, so $A_{120} = 0$: $$300\,000(1.003)^{120} = R \cdot \frac{(1.003)^{120} - 1}{0.003}.$$ Now $(1.003)^{120} \approx 1.431405$. $$R = \frac{300\,000 \times 0.003 \times 1.431405}{1.431405 - 1} = \frac{1288.26}{0.431405} \approx 2986.2.$$ Hence $R \approx \$2\,986$ per month (to the nearest dollar).	7	
6.	**(a)** 0.58 **(b)** (0.5316, 0.6284) **(c) the whole interval lies above** 0.5**, so the claim is supported (d)** $n \geq 1040$ (a) $\hat{p} = \frac{232}{400} = 0.58$. (b) Standard error $= \sqrt{\frac{0.58 \times 0.42}{400}} = \sqrt{\frac{0.2436}{400}} = \sqrt{0.000609} \approx 0.024678$. Margin of error $= 1.96 \times 0.024678 \approx 0.04837$. Interval: $(0.58 - 0.0484,\ 0.58 + 0.0484) \approx (0.5316,\ 0.6284)$. (c) Since the lower limit $0.5316 > 0.5$, every value in the 95% confidence interval exceeds 0.5. The data are consistent with the claim that more than half of the voters support the proposal. (d) Require $1.96\sqrt{\frac{0.58 \times 0.42}{n}} \leq 0.03$: $$\sqrt{\frac{0.2436}{n}} \leq \frac{0.03}{1.96} \approx 0.015306,$$ $$\frac{0.2436}{n} \leq 0.00023428 \Rightarrow n \geq \frac{0.2436}{0.00023428} \approx 1039.8.$$ Hence the minimum sample size is 1040.	8	
7.	**(a)** $BD = 5$ **cm (b)** 84 cm^2 **(c)** $BP = 6$ **cm (d)** $r = 4$ **cm** (a) Let $BD = x$, so $DC = 14 - x$. By Pythagoras in $\triangle ABD$ and $\triangle ACD$: $$AD^2 = 13^2 - x^2 = 15^2 - (14 - x)^2.$$ $$169 - x^2 = 225 - 196 + 28x - x^2 \Rightarrow 169 = 29 + 28x \Rightarrow x = 5.$$ So $BD = 5$ cm. (b) $AD = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$ cm. Area $= \frac{1}{2} \times 14 \times 12 = 84$ cm^2. (c) Tangents from an external point are equal. Let $BP = s - a$ where s is the semi-perimeter: $s = \frac{13 + 14 + 15}{2} = 21$. Then $BP = s - b$ with $b = CA = 15$: $BP = 21 - 15 = 6$ cm. (Check: $BP = s - AC = 6$, $PC = s - AB = 8$, and $BP + PC = 14 = BC$.) (d) Area $= rs$, so $84 = r \times 21$, giving $r = 4$ cm.	9	
SECTION B (10 marks)			

TUTGATHER

Question	Solution	Marks	Remarks
8.	**(a)** $P(x) = -0.03x^2 + 50x - 18\,000$ **(b)** $x \approx 833$ **units, maximum profit** $\approx \$2\,833$ **(c)** $403 \le x \le 1264$ **(d) not possible, since the maximum profit** $\approx \$2\,833 < \$20\,000$ (a) $P(x) = (90x - 0.01x^2) - (0.02x^2 + 40x + 18\,000) = -0.03x^2 + 50x - 18\,000$. (b) This is a downward-opening parabola. The maximum occurs at $$x = -\frac{b}{2a} = -\frac{50}{2(-0.03)} = \frac{50}{0.06} \approx 833.3.$$ Since x must be a whole number of units, take $x = 833$ (or 834; both give essentially the same value). Then $$P(833) = -0.03(833)^2 + 50(833) - 18\,000 \approx -20\,816.7 + 41\,650 - 18\,000 \approx 2\,833.3.$$ So the maximum weekly profit is about \$2 833 when $x \approx 833$ units. (c) Break even when $P(x) = 0$: $-0.03x^2 + 50x - 18\,000 = 0$, i.e. $0.03x^2 - 50x + 18\,000 = 0$, or $3x^2 - 5000x + 1\,800\,000 = 0$. $$x = \frac{5000 \pm \sqrt{5000^2 - 4(3)(1\,800\,000)}}{2(3)} = \frac{5000 \pm \sqrt{25\,000\,000 - 21\,600\,000}}{6} = \frac{5000 \pm \sqrt{3\,400\,000}}{6}.$$ $\sqrt{3\,400\,000} \approx 1843.9$, so $x \approx \frac{5000 \pm 1843.9}{6}$, giving $x \approx 526.0$ or $x \approx 1140.7$. Since P is a downward parabola, $P(x) \ge 0$ between the roots: $526 \le x \le 1141$ (to the nearest integer). (d) The maximum possible weekly profit is about \$2 833, which is far below \$20 000. Hence a weekly profit of at least \$20 000 is impossible under this model; there is no such range of x.	10	

TUTGATHER