MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: More about Probability · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$\frac{7}{40}$ Let P be the set of students studying Physics and C be the set studying Chemistry. We have $n(P) = 25$, $n(C) = 18$, $n(P \cap C) = 10$. The number studying at least one subject is $n(P \cup C) = n(P) + n(C) - n(P \cap C) = 25 + 18 - 10 = 33$. Hence the number studying neither is $40 - 33 = 7$. The required probability is $\frac{7}{40}$.	3	
2.	$P(A \cap B) = 0.2$**,** $P(A' \cap B') = 0.3$ By the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Thus $0.7 = 0.5 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.2$. Also $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.7 = 0.3$.	3	
3.	**(a) No, because** $A \cap B = \{6\} \neq \varnothing$**. (b)** $\frac{2}{3}$ (a) $A = \{2, 4, 6\}$ and $B = \{3, 6\}$. Since $6 \in A \cap B$, the events are not mutually exclusive. (b) $P(A) = \frac{3}{6}$, $P(B) = \frac{2}{6}$, $P(A \cap B) = \frac{1}{6}$. Hence $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.	4	
4.	**(a)** 0.18 **(b)** 0.54 **(c)** 0.72 Let R_X and R_Y be the events that spinners X and Y land on red respectively. Since the spinners are independent, $P(R_X \cap R_Y) = P(R_X)P(R_Y) = 0.3 \times 0.6 = 0.18$. (b) Exactly one red means $(R_X \cap R_Y') \cup (R_X' \cap R_Y)$. These are mutually exclusive, so probability $= 0.3 \times 0.4 + 0.7 \times 0.6 = 0.12 + 0.42 = 0.54$. (c) $P(R_X \cup R_Y) = 1 - P(R_X' \cap R_Y') = 1 - 0.7 \times 0.4 = 1 - 0.28 = 0.72$.	6	
SECTION A(2) (24 marks)			
5.	**(a)** $\frac{3}{7}$ **(b)** $\frac{3}{28}$ **(c)** $\frac{3}{8}$ Let R_i and B_i denote the event that the i-th ball is red and blue respectively. (a) After drawing a red ball, 7 balls remain of which 3 are blue, so $P(B_2 \mid R_1) = \frac{3}{7}$. (b) $P(B_1 \cap B_2) = P(B_1)P(B_2 \mid B_1) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$. (c) By the law of total probability, $P(B_2) = P(R_1 \cap B_2) + P(B_1 \cap B_2) = P(R_1)P(B_2 \mid R_1) + P(B_1)P(B_2 \mid B_1) = \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{2}{7} = \frac{15}{56} + \frac{6}{56} = \frac{21}{56} = \frac{3}{8}$.	7	
6.	**(a) Venn diagram with regions:** M **only** 30**,** P **only** 15**, both** 30**, neither** 25**. (b)** $\frac{9}{20}$ **(c)** $\frac{1}{2}$ (a) Let M and P denote the sets of students who like Mathematics and Physics. $n(M \cap P) = 30$, $n(M \setminus P) = 60 - 30 = 30$, $n(P \setminus M) = 45 - 30 = 15$, $n((M \cup P)') = 100 - (30 + 30 + 15) = 25$. (b) Exactly one subject: $30 + 15 = 45$ students, probability $\frac{45}{100} = \frac{9}{20}$. (c) $P(P \mid M) = \frac{n(M \cap P)}{n(M)} = \frac{30}{60} = \frac{1}{2}$.	8	

TUTGATHER

Question	Solution	Marks	Remarks
7.	**(a)** 0.025 **(b)** 0.055 **(c)** $\frac{5}{11}$ Let R be the event of rain and L the event of being late. $P(R) = 0.25$, $P(L \mid R) = 0.10$, $P(L \mid R') = 0.04$. (a) $P(R \cap L) = P(R)P(L \mid R) = 0.25 \times 0.10 = 0.025$. (b) $P(L) = P(R \cap L) + P(R' \cap L) = 0.025 + 0.75 \times 0.04 = 0.025 + 0.03 = 0.055$. (c) $P(R \mid L) = \frac{P(R \cap L)}{P(L)} = \frac{0.025}{0.055} = \frac{25}{55} = \frac{5}{11}$.	9	
SECTION B (10 marks)			
8.	**(a)** $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$, $P(C) = \frac{1}{2}$. **(b)** $P(A \cup B) = \frac{7}{8}$. **(c) Not independent since** $P(A \cap C) = \frac{3}{8} \neq P(A)P(C) = \frac{1}{4}$ **. (d)** $P(A \cup C) = \frac{5}{8}$. The sample space is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$, each outcome probability $\frac{1}{8}$. (a) $A = \{HHH, HHT, HTH, THH\}$, so $P(A) = \frac{4}{8} = \frac{1}{2}$. $B = \{HTT, THT, TTH\}$, so $P(B) = \frac{3}{8}$. $C = \{HHH, HHT, HTH, HTT\}$, so $P(C) = \frac{4}{8} = \frac{1}{2}$. (b) $A \cap B = \varnothing$, so they are mutually exclusive. $P(A \cup B) = P(A) + P(B) = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$. (c) $A \cap C = \{HHH, HHT, HTH\}$, so $P(A \cap C) = \frac{3}{8}$. But $P(A)P(C) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Since $\frac{3}{8} \neq \frac{1}{4}$, A and C are not independent. (d) $P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{1}{2} + \frac{1}{2} - \frac{3}{8} = \frac{5}{8}$.	10	

TUTGATHER