MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: Loci · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**The locus is the perpendicular bisector of AB; its equation is $x + y - 7 = 0$.** The locus of points equidistant from two fixed points A and B is the perpendicular bisector of the line segment AB. The midpoint of AB is $M(\frac{1+5}{2}, \frac{2+6}{2}) = (3, 4)$. The slope of AB is $\frac{6-2}{5-1} = 1$, so the perpendicular bisector has slope -1. Hence its equation is $y - 4 = -1(x - 3)$, i.e. $x + y - 7 = 0$.	3	
2.	$(x - 2)^2 + (y + 1)^2 = 25$ The locus of points at a fixed distance r from a fixed point C is a circle with centre C and radius r. Here $C = (2, -1)$ and $r = 5$. Thus the equation is $(x - 2)^2 + (y + 1)^2 = 5^2$, i.e. $(x - 2)^2 + (y + 1)^2 = 25$.	3	
3.	$3x + 4y - 20 = 0$ **or** $3x + 4y = 0$ The distance from $P(x, y)$ to L is $\frac{\lvert 3x + 4y - 10 \rvert}{\sqrt{3^2 + 4^2}} = \frac{\lvert 3x + 4y - 10 \rvert}{5}$. Setting this equal to 2 gives $\lvert 3x + 4y - 10 \rvert = 10$. Hence $3x + 4y - 10 = 10$ or $3x + 4y - 10 = -10$. So the loci are the parallel lines $3x + 4y - 20 = 0$ and $3x + 4y = 0$.	4	
4.	**(a) Centre** $(3, 4)$**, radius** 3**. (b)** $(1, 3)$ **and** $(4, 0)$**.** (a) Complete the square: $x^2 - 6x + y^2 - 8y + 16 = 0$ $\Rightarrow (x - 3)^2 - 9 + (y - 4)^2 - 16 + 16 = 0$ $\Rightarrow (x - 3)^2 + (y - 4)^2 = 9$. So the centre is $(3, 4)$ and the radius is 3. (b) From Γ_2, $y = 4 - x$. Substitute into Γ_1: $x^2 + (4 - x)^2 - 6x - 8(4 - x) + 16 = 0$ $\Rightarrow x^2 + 16 - 8x + x^2 - 6x - 32 + 8x + 16 = 0$ $\Rightarrow 2x^2 - 6x = 0$ $\Rightarrow 2x(x - 3) = 0$, so $x = 0$ or $x = 3$. When $x = 0$, $y = 4$; when $x = 3$, $y = 1$. Thus the intersection points are $(0, 4)$ and $(3, 1)$.	6	

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Question	Solution	Marks	Remarks
SECTION A(2) (24 marks)			
5.	**(a)** $3x + 2y - 9 = 0$**. (b)** $\frac{27}{4}$ **square units.** (a) $PA^2 = (x+2)^2 + (y-1)^2$ and $PB^2 = (x-4)^2 + (y-5)^2$. $PA = PB \Rightarrow PA^2 = PB^2$: $(x+2)^2 + (y-1)^2 = (x-4)^2 + (y-5)^2$ $x^2 + 4x + 4 + y^2 - 2y + 1 = x^2 - 8x + 16 + y^2 - 10y + 25$ $4x - 2y + 5 = -8x - 10y + 41$ $12x + 8y - 36 = 0$ $3x + 2y - 9 = 0$. (b) For C, set $y = 0$: $3x - 9 = 0 \Rightarrow x = 3$, so $C(3, 0)$. For D, set $x = 0$: $2y - 9 = 0 \Rightarrow y = \frac{9}{2}$, so $D(0, \frac{9}{2})$. Area of $\triangle OCD = \frac{1}{2} \cdot OC \cdot OD = \frac{1}{2} \cdot 3 \cdot \frac{9}{2} = \frac{27}{4}$ square units.	7	
6.	**(a)** $x^2 + y^2 - 10x + 4y + 19 = 0$**, a circle. (b) Centre** $(5, -2)$**, radius** $\sqrt{10}$**.** (a) The condition is $PA = 2PB$, so $PA^2 = 4PB^2$. $(x-1)^2 + (y-2)^2 = 4[(x-4)^2 + (y+1)^2]$ $x^2 - 2x + 1 + y^2 - 4y + 4 = 4(x^2 - 8x + 16 + y^2 + 2y + 1)$ $x^2 + y^2 - 2x - 4y + 5 = 4x^2 + 4y^2 - 32x + 8y + 68$ $0 = 3x^2 + 3y^2 - 30x + 12y + 63$ Divide by 3: $x^2 + y^2 - 10x + 4y + 21 = 0$. This is of the form $x^2 + y^2 + Dx + Ey + F = 0$ with $D^2 + E^2 - 4F = 100 + 16 - 84 = 32 > 0$, so it represents a circle. (b) Complete the square: $(x-5)^2 - 25 + (y+2)^2 - 4 + 21 = 0$ $(x-5)^2 + (y+2)^2 = 8$. Thus the centre is $(5, -2)$ and the radius is $\sqrt{8} = 2\sqrt{2}$.	8	
7.	**(a)** $3x + 4y - 24 = 0$**. (b)** $3x - y - 24 = 0$ **and** $3x + 9y - 24 = 0$**. (c) They are the internal and external angle bisectors of the angle formed by lines** AB **and** OA **(the** x**-axis) at** A**.** (a) The line through $A(0, 6)$ and $B(8, 0)$ has slope $\frac{0-6}{8-0} = -\frac{3}{4}$. Using point-slope form with A: $y - 6 = -\frac{3}{4}(x - 0)$, i.e. $3x + 4y - 24 = 0$. (b) The distance from $P(x, y)$ to AB is $\frac{\lvert 3x+4y-24 \rvert}{\sqrt{3^2+4^2}} = \frac{\lvert 3x+4y-24 \rvert}{5}$. The distance from P to the x-axis is $\lvert y \rvert$. Equating: $\frac{\lvert 3x+4y-24 \rvert}{5} = \lvert y \rvert \Rightarrow \lvert 3x + 4y - 24 \rvert = 5 \lvert y \rvert$. Hence $3x + 4y - 24 = 5y$ or $3x + 4y - 24 = -5y$. These simplify to $3x - y - 24 = 0$ and $3x + 9y - 24 = 0$. Thus the locus consists of the two straight lines $3x - y - 24 = 0$ and $3x + 9y - 24 = 0$. (c) The two lines are the angle bisectors of the angle between the line AB and the x-axis (which is the line OA). One bisects the internal angle at A and the other bisects the external angle at A.	9	
SECTION B (10 marks)			

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Question	Solution	Marks	Remarks
8.	**(a)** Γ_1 **: centre** $(2, 3)$**, radius** 2**;** Γ_2 **: centre** $(5, 1)$**, radius** $\sqrt{5}$**. (b)** $(2, 1)$ **and** $(4, 3)$**. (c)** $3x - 2y - 6 = 0$**. (d)** $3x - 2y - 6 = 0$**.** (a) For Γ_1 : $x^2 - 4x + y^2 - 6y + 9 = 0$ $\Rightarrow (x-2)^2 - 4 + (y-3)^2 - 9 + 9 = 0$ $\Rightarrow (x-2)^2 + (y-3)^2 = 4$. Centre $C_1(2, 3)$, radius $r_1 = 2$. For Γ_2 : $x^2 - 10x + y^2 - 2y + 21 = 0$ $\Rightarrow (x-5)^2 - 25 + (y-1)^2 - 1 + 21 = 0$ $\Rightarrow (x-5)^2 + (y-1)^2 = 5$. Centre $C_2(5, 1)$, radius $r_2 = \sqrt{5}$. (b) Subtract the equations: $(x^2 + y^2 - 4x - 6y + 9) - (x^2 + y^2 - 10x - 2y + 21) = 0$ $6x - 4y - 12 = 0 \Rightarrow 3x - 2y - 6 = 0$. From this, $y = \frac{3x-6}{2}$. Substitute into Γ_1 : $x^2 + (\frac{3x-6}{2})^2 - 4x - 6(\frac{3x-6}{2}) + 9 = 0$ Multiply by 4: $4x^2 + (3x-6)^2 - 16x - 12(3x-6) + 36 = 0$ $4x^2 + 9x^2 - 36x + 36 - 16x - 36x + 72 + 36 = 0$ $13x^2 - 88x + 144 = 0$ Discriminant $= 88^2 - 4(13)(144) = 7744 - 7488 = 256$, so $x = \frac{88 \pm 16}{26}$. $x = 4$ or $x = \frac{72}{26} = \frac{36}{13}$. When $x = 4$, $y = \frac{12-6}{2} = 3$. When $x = \frac{36}{13}$, $y = \frac{\frac{108}{13} - 6}{2} = \frac{\frac{30}{13}}{2} = \frac{15}{13}$. Thus the intersection points are $(4, 3)$ and $(\frac{36}{13}, \frac{15}{13})$. (c) The power of $P(x, y)$ with respect to Γ_1 is $x^2 + y^2 - 4x - 6y + 9$. The power with respect to Γ_2 is $x^2 + y^2 - 10x - 2y + 21$. Equating: $x^2 + y^2 - 4x - 6y + 9 = x^2 + y^2 - 10x - 2y + 21$ $6x - 4y - 12 = 0 \Rightarrow 3x - 2y - 6 = 0$. This is a straight line, so the locus is the line $3x - 2y - 6 = 0$. (d) The common chord is the line through the intersection points of the two circles. From (b), the intersection points satisfy $3x - 2y - 6 = 0$, so the common chord has equation $3x - 2y - 6 = 0$. This is the same as the locus found in (c).	10	

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