MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: More about Graphs of Functions · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**(a)** $f(x) = (x-2)^2 - 1$ **(b) Vertex** $(2, -1)$**, a minimum point.** (a) Complete the square: $f(x) = x^2 - 4x + 3 = (x^2 - 4x + 4) - 4 + 3 = (x-2)^2 - 1$. (b) From (a), $f(x) = (x-2)^2 - 1 \geq -1$ for all real x, with equality when $x = 2$. Hence the vertex is $(2, -1)$ and it is a minimum point (the coefficient of x^2 is positive, so the parabola opens upwards).	3	
2.	**(a)** $(-1, 5)$ **(b)** $(3, -5)$ **(c)** $(3, 3)$ (a) Translating left by 4 replaces x by $x + 4$, so a point (x, y) maps to $(x-4, y)$: $(3, 5) \mapsto (-1, 5)$. (b) Reflection in the x-axis sends (x, y) to $(x, -y)$: $(3, 5) \mapsto (3, -5)$. (c) Translating down by 2 sends (x, y) to $(x, y-2)$: $(3, 5) \mapsto (3, 3)$.	3	
3.	**(a) Vertical dilation by factor 3, then translate 1 unit left and 6 units down.** **(b)** $(-1, -6)$ (a) Starting from $y = x^2$: - multiply the function by 3: $y = 3x^2$ (vertical dilation, factor 3, parallel to the y-axis); - replace x by $x + 1$: $y = 3(x+1)^2$ (translate 1 unit to the left); - subtract 6: $y = 3(x+1)^2 - 6$ (translate 6 units down). (b) Since $(x+1)^2 \geq 0$, the minimum value is -6, attained when $x = -1$. So the vertex is $(-1, -6)$.	4	

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Question	Solution	Marks	Remarks
4.	**(a) f: vertex $(1, -1)$; g: vertex $(1, 9)$. (b) Reflect in the x-axis, then translate 8 units up. (c) Maximum value 9, at $x = 1$.** (a) $f(x) = x^2 - 2x = (x-1)^2 - 1$, so the vertex is $(1, -1)$. $g(x) = -x^2 + 2x + 8 = -(x^2 - 2x) + 8 = -(x-1)^2 + 1 + 8 = -(x-1)^2 + 9$, so the vertex is $(1, 9)$. (b) Note $g(x) = -f(x) + 8$. Reflecting $y = f(x)$ in the x-axis gives $y = -f(x)$; translating the result 8 units up gives $y = -f(x) + 8 = g(x)$. (Equivalently, the vertex $(1, -1)$ maps to $(1, 1)$ then to $(1, 9)$, consistent with (a).) (c) Since $-(x-1)^2 \le 0$, we have $g(x) \le 9$, with equality when $x = 1$. Hence the maximum value of $g(x)$ is 9, occurring at $x = 1$.	6	
SECTION A(2) (24 marks)			
5.	**(a) x-intercepts 1 and 5; y-intercept 5. (b) Vertex $(3, -4)$. (d) $1 < x < 5$** (a) $f(x) = 0$: $x^2 - 6x + 5 = (x-1)(x-5) = 0$, so $x = 1$ or $x = 5$. The x-intercepts are 1 and 5. $f(0) = 5$, so the y-intercept is 5. (b) $f(x) = x^2 - 6x + 5 = (x^2 - 6x + 9) - 9 + 5 = (x-3)^2 - 4$. The vertex is $(3, -4)$. (c) The parabola opens upwards (coefficient of x^2 is $1 > 0$), with vertex $(3, -4)$, passing through $(1, 0)$, $(5, 0)$ and $(0, 5)$; the axis of symmetry is $x = 3$. The sketch shows a U-shaped curve with these four marked points. (d) The graph lies below the x-axis strictly between the two x-intercepts, so $f(x) < 0$ exactly when $1 < x < 5$.	7	
6.	**(a) $a = -1$ (b) $h_2(x) = (x-7)^2 - 3$ (c) Vertex $(7, -3)$, a minimum point. (d)** 46 (a) $h(0) = -1$: $-1 = a(0-2)^2 + 3 = 4a + 3$, so $4a = -4$ and $a = -1$. Hence $h(x) = -(x-2)^2 + 3$. (b) Reflection in the x-axis: $h_1(x) = -h(x) = (x-2)^2 - 3$. Translate 5 units right (replace x by $x - 5$): $h_2(x) = ((x-5)-2)^2 - 3 = (x-7)^2 - 3$. (c) From (b), the vertex is $(7, -3)$. Since the coefficient of $(x-7)^2$ is $1 > 0$, the graph opens upwards and the vertex is a minimum point. (d) $h_2(0) = (0-7)^2 - 3 = 49 - 3 = 46$. The y-intercept is 46.	8	

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Question	Solution	Marks	Remarks
7.	**(a) shown (b) Vertical dilation factor 4, then translate 2 units right and 3 units down. (c)** $k(x) = 2(x-2)^2 - \frac{3}{2}$**, minimum** $-\frac{3}{2}$**. (d)** $c > -3$ (a) $4(x-2)^2 - 3 = 4(x^2 - 4x + 4) - 3 = 4x^2 - 16x + 16 - 3 = 4x^2 - 16x + 13 = g(x)$. (b) From $y = x^2$: multiply by 4 to get $y = 4x^2$ (vertical dilation, factor 4); replace x by $x - 2$ to get $y = 4(x-2)^2$ (translate 2 units right); subtract 3 to get $y = 4(x-2)^2 - 3$ (translate 3 units down). (c) $k(x) = \frac{1}{2}g(x) = \frac{1}{2}[4(x-2)^2 - 3] = 2(x-2)^2 - \frac{3}{2}$. Since $2(x-2)^2 \geq 0$, the minimum value of $k(x)$ is $-\frac{3}{2}$ (at $x = 2$). (d) The minimum value of g is -3 (at $x = 2$), and $g(x) \to \infty$ as $x \to \pm\infty$. A horizontal line $y = c$ meets the parabola twice exactly when c is greater than the minimum value, i.e. $c > -3$.	9	
SECTION B (10 marks)			
8.	**(a)** $f(x) = (x-2)^2 + 3$**, vertex** $(2, 3)$**, minimum 3;** $g(x) = -(x-2)^2 + 5$**, vertex** $(2, 5)$**, maximum 5. (b) Reflection in the** x**-axis followed by a translation 6 units up. (c)** $(1, 4)$ **and** $(3, 4)$**. (d) Claim false;** $f(x) < g(x)$ **exactly when** $1 < x < 3$**. (e)** $t = \sqrt{2}$ (a) $f(x) = x^2 - 4x + 7 = (x-2)^2 - 4 + 7 = (x-2)^2 + 3$: vertex $(2, 3)$, minimum value 3. $g(x) = -x^2 + 4x + 1 = -(x^2 - 4x) + 1 = -(x-2)^2 + 4 + 1 = -(x-2)^2 + 5$: vertex $(2, 5)$, maximum value 5. (b) $6 - f(x) = 6 - [(x-2)^2 + 3] = 3 - (x-2)^2 = -(x-2)^2 + 3$. But $g(x) = -(x-2)^2 + 5$, so this is not equal to $g(x)$; instead $g(x) = 8 - f(x)$ since $8 - f(x) = 8 - (x-2)^2 - 3 = 5 - (x-2)^2 = g(x)$. Hence $g(x) = 8 - f(x)$: reflect the graph of $y = f(x)$ in the x-axis (giving $y = -f(x)$) and then translate 8 units upwards. (c) At an intersection, $f(x) = g(x)$: $x^2 - 4x + 7 = -x^2 + 4x + 1$, so $2x^2 - 8x + 6 = 0$, i.e. $x^2 - 4x + 3 = 0$, giving $(x-1)(x-3) = 0$, so $x = 1$ or $x = 3$. Then $f(1) = 1 - 4 + 7 = 4$ and $f(3) = 9 - 12 + 7 = 4$. The intersection points are $(1, 4)$ and $(3, 4)$. (d) $f(x) - g(x) = 2x^2 - 8x + 6 = 2(x-1)(x-3)$. This is negative exactly when $1 < x < 3$. So the claim is false; $f(x) < g(x)$ precisely for $1 < x < 3$ (and $f(x) = g(x)$ at $x = 1, 3$, with $f(x) > g(x)$ otherwise). (e) Translating t units right replaces x by $x - t$: $F(x) = f(x - t) = (x - t - 2)^2 + 3$, whose minimum value is 3 for every t. This can never equal the minimum value of g, which is 5. Hence no such t exists; the required value does not exist.	10	

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