MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: Arithmetic and Geometric Sequences and their Summations · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**(a)** $a_1 = -2$**,** $a_4 = 43$**. (b) No;** 202 **is not a term.** (a) $a_1 = 3(1)^2 - 5 = 3 - 5 = -2$. $a_4 = 3(4)^2 - 5 = 48 - 5 = 43$. (b) Suppose $a_n = 202$. Then $3n^2 - 5 = 202$, so $3n^2 = 207$, giving $n^2 = 69$. Since n must be a positive integer and $\sqrt{69} \approx 8.31$ is not an integer, no positive integer n satisfies the equation. Hence 202 is not a term of the sequence.	3	
2.	**(a) first term** $= 1$**, common difference** $= 4$**. (b)** $T_{30} = 117$**.** Let the first term be a and the common difference be d. (a) $T_5 = a + 4d = 17$ and $T_{12} = a + 11d = 45$. Subtracting: $7d = 28$, so $d = 4$. Then $a + 4(4) = 17$, so $a = 1$. (b) $T_{30} = a + 29d = 1 + 29(4) = 1 + 116 = 117$.	3	
3.	**(a)** $r = 3$**,** $a = 2$**. (b)** $S_8 = 6560$**.** Let the first term be a and the common ratio be r. (a) $T_2 = ar = 6$ and $T_5 = ar^4 = 162$. Dividing: $\frac{ar^4}{ar} = \frac{162}{6}$, so $r^3 = 27$, giving $r = 3$ (positive terms). Then $a(3) = 6$, so $a = 2$. (b) $S_8 = \frac{a(r^8 - 1)}{r - 1} = \frac{2(3^8 - 1)}{3 - 1} = \frac{2(6561 - 1)}{2} = 6560$.	4	

TUTGATHER

Question	Solution	Marks	Remarks
4.	**(a)** 80 **terms. (b)** 16360**. (c)** 5400**.** The series is arithmetic with first term $a = 7$ and common difference $d = 5$. (a) $T_n = 7 + (n-1)(5) = 5n + 2$. Setting $5n + 2 = 402$ gives $5n = 400$, so $n = 80$. (b) $S_{80} = \frac{80}{2}(7 + 402) = 40 \times 409 = 16360$. (c) A term $5n + 2$ is divisible by 3 when $5n + 2 \equiv 0 \pmod 3$, i.e. $2n + 2 \equiv 0 \pmod 3$, i.e. $n \equiv 2 \pmod 3$. So $n = 2, 5, 8, \ldots, 80$. These form an arithmetic sequence with first term 2, common difference 3 and last term 80: number of terms $= \frac{80-2}{3} + 1 = 27$. The corresponding terms are $T_2 = 12$ and $T_{80} = 402$. Sum $= \frac{27}{2}(12 + 402) = \frac{27}{2}(414) = 27 \times 207 = 5589$. Recheck: terms divisible by 3 are $12, 27, 42, \ldots, 402$; common difference 15; number $= \frac{402-12}{15} + 1 = 27$; sum $= \frac{27}{2}(12 + 402) = 5589$.	6	
SECTION A(2) (24 marks)			
5.	**(a)** \$7117 **(nearest dollar). (b)** \$75 129 **(nearest dollar). (c)** $n = 42$**.** The monthly deposits form a geometric sequence with first term $a = 5000$ and common ratio $r = 1.04$. (a) $T_{10} = 5000(1.04)^9 \approx 5000 \times 1.423312 \approx 7116.56$, i.e. \$7117 (nearest dollar). (b) $S_{12} = \frac{5000(1.04^{12} - 1)}{1.04 - 1} = \frac{5000(1.601032 - 1)}{0.04} \approx \frac{5000 \times 0.601032}{0.04} \approx 75129.0$, i.e. \$75 129 (nearest dollar). (c) Solve $\frac{5000(1.04^n - 1)}{0.04} > 1\,000\,000$: $1.04^n - 1 > \frac{1\,000\,000 \times 0.04}{5000} = 8$, so $1.04^n > 9$. Taking logarithms: $n > \frac{\log 9}{\log 1.04} \approx \frac{0.954243}{0.017033} \approx 56.02$. So the least value is $n = 57$. Check: $S_{56} \approx \frac{5000(1.04^{56} - 1)}{0.04} \approx 5000 \times \frac{8.9927}{0.04}$ · — recompute: $1.04^{56} \approx 9.0$; $S_{56} \approx \frac{5000 \times 8.0}{0.04} = 1\,000\,000$ boundary; $S_{57} > 1\,000\,000$. Hence $n = 57$.	7	
6.	**(a)** $T_1 = 4$**,** $T_2 = 6$**. (b) shown. (c) arithmetic with** $a = 4$**,** $d = 2$**; sum** $= 616$**.** (a) $T_1 = S_1 = 1^2 + 3(1) = 4$. $S_2 = 2^2 + 3(2) = 10$, so $T_2 = S_2 - S_1 = 10 - 4 = 6$. (b) For $n \geq 2$, $T_n = S_n - S_{n-1} = (n^2 + 3n) - [(n-1)^2 + 3(n-1)]$. $(n-1)^2 + 3(n-1) = n^2 - 2n + 1 + 3n - 3 = n^2 + n - 2$. So $T_n = (n^2 + 3n) - (n^2 + n - 2) = 2n + 2$. For $n = 1$: $2(1) + 2 = 4 = T_1$. Hence $T_n = 2n + 2$ for all $n \geq 1$. (c) $T_{n+1} - T_n = [2(n+1) + 2] - (2n + 2) = 2$, a constant, so the sequence is arithmetic with first term 4 and common difference 2. Sum from T_{10} to T_{25} $= S_{25} - S_9 = (25^2 + 3 \cdot 25) - (9^2 + 3 \cdot 9) = (625 + 75) - (81 + 27) = 700 - 108 = 592$. Check by formula: $T_{10} = 22$, $T_{25} = 52$, number of terms $= 16$; sum $= \frac{16}{2}(22 + 52) = 8 \times 74 = 592$.	8	

TUTGATHER

Question	Solution	Marks	Remarks
7.	**(a)** $T_n = 6n + 2$**,** $S_n = n(3n + 5)$**. (b)** $n = 25$**. (c) arithmetic with first term** 20**, common difference** 18**; sum** = 3820**.** (a) $T_n = 8 + (n - 1)6 = 6n + 2$. $S_n = \frac{n}{2}[2(8) + (n - 1)6] = \frac{n}{2}(16 + 6n - 6) = \frac{n}{2}(6n + 10) = n(3n + 5)$. (b) $n(3n + 5) > 2027$, i.e. $3n^2 + 5n - 2027 > 0$. Solving $3n^2 + 5n - 2027 = 0$: $n = \frac{-5 \pm \sqrt{25 + 4(3)(2027)}}{6} = \frac{-5 \pm \sqrt{24349}}{6}$. $\sqrt{24349} \approx 156.04$, so $n \approx \frac{151.04}{6} \approx 25.17$. Since n is a positive integer, the least value is $n = 25$. Check: $S_{24} = 24(72 + 5) = 24 \times 77 = 1848 \le 2027$; $S_{25} = 25(75 + 5) = 25 \times 80 = 2000$. Recompute: $S_{25} = 25(3 \cdot 25 + 5) = 25 \times 80 = 2000 < 2027$; $S_{26} = 26(78 + 5) = 26 \times 83 = 2158 > 2027$. Hence $n = 26$. (c) The new sequence has terms $U_k = T_{3k} = 6(3k) + 2 = 18k + 2$. $U_{k+1} - U_k = 18$, a constant, so it is arithmetic with first term $U_1 = 20$ and common difference 18. $S_{20} = \frac{20}{2}[2(20) + 19(18)] = 10(40 + 342) = 10 \times 382 = 3820$.	9	
SECTION B (10 marks)			
8.	**(a)** $a = 8$**,** $r = \frac{1}{2}$**. (b)** $n = 8$**. (c) second sum** $= \frac{32}{3}$**; difference** $= \frac{16}{3}$**.** (a) $S_\infty = \frac{a}{1 - r} = 16$, so $a = 16(1 - r)$. Sum of first two terms: $a + ar = a(1 + r) = 12$. Substituting: $16(1 - r)(1 + r) = 12$, so $16(1 - r^2) = 12$, giving $1 - r^2 = \frac{3}{4}$, hence $r^2 = \frac{1}{4}$. Since $0 < r < 1$, $r = \frac{1}{2}$. Then $a = 16(1 - \frac{1}{2}) = 8$. (b) $S_n = \frac{8(1 - (\frac{1}{2})^n)}{1 - \frac{1}{2}} = 16(1 - (\frac{1}{2})^n)$. Require $16(1 - (\frac{1}{2})^n) > 15.9$, i.e. $1 - (\frac{1}{2})^n > 0.99375$, so $(\frac{1}{2})^n < 0.00625 = \frac{1}{160}$. Thus $2^n > 160$. Since $2^7 = 128 < 160$ and $2^8 = 256 > 160$, the least value is $n = 8$. (c) Second sequence: first term 8, common ratio $r^2 = \frac{1}{4}$. Sum to infinity $= \frac{8}{1 - \frac{1}{4}} = \frac{8}{\frac{3}{4}} = \frac{32}{3}$. Difference $= \frac{32}{3} - 16 = \frac{32}{3} - \frac{48}{3} = -\frac{16}{3}$, i.e. a difference of $\frac{16}{3}$ in magnitude.	10	

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