MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: Variations · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$x = 9$ Let $y = kx^2$, where k is a non-zero constant. Substituting $x = 4$ and $y = 48$: $48 = k(4)^2 = 16k$, so $k = 3$. Hence $y = 3x^2$. When $y = 243$: $3x^2 = 243$, so $x^2 = 81$ and $x = \pm 9$. Since $x > 0$, $x = 9$.	3	
2.	**(a)** $y = 15$ Let $y = \frac{k}{x^{1/2}}$. Substituting $x = 9$ and $y = 20$: $20 = \frac{k}{9^{1/2}} = \frac{k}{3}$, so $k = 60$. Hence $y = \frac{60}{x^{1/2}}$. When $x = 16$: $y = \frac{60}{16^{1/2}} = \frac{60}{4} = 15$.	2	
	(b) $x = 36$ Set $\frac{60}{x^{1/2}} = 10$. Then $x^{1/2} = 6$, so $x = 36$.	1	
3.	**(a)** $z = 3x^2 y$ Let $z = kx^2 y$. Substituting $x = 2$, $y = 3$ and $z = 36$: $36 = k(2)^2(3) = 12k$, so $k = 3$. Hence $z = 3x^2 y$.	2	
	(b) $z = 135$ Substituting $x = 3$ and $y = 5$ into $z = 3x^2 y$: $z = 3(3)^2(5) = 3 \times 9 \times 5 = 135$.	2	
4.	**(a)** $T = \frac{240}{n}$ Let $T = \frac{k}{n}$. Substituting $n = 3$ and $T = 80$: $80 = \frac{k}{3}$, so $k = 240$. Hence $T = \frac{240}{n}$.	2	

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Question	Solution	Marks	Remarks
	(b) 48 minutes When $n = 5$: $T = \frac{240}{5} = 48$. The pool is filled in 48 minutes.	2	
	(c) 6 pumps Require $\frac{240}{n} \le 45$, so $n \ge \frac{240}{45} = \frac{16}{3} = 5.33\ldots$. Since n must be a positive integer, the least possible value is $n = 6$. With 6 pumps, $T = 40$ minutes, which is within 45 minutes.	2	
SECTION A(2) (24 marks)			
5.	**(a)** 9375 dollars Let $C = kw^3$. Substituting $w = 2$ and $C = 4800$: $4800 = k(2)^3 = 8k$, so $k = 600$. Hence $C = 600w^3$. When $w = 2.5$: $C = 600(2.5)^3 = 600 \times 15.625 = 9375$. The cost is 9375 dollars.	2	
	(b) 72.8% Let the original width be w, so the original cost is $C = 600w^3$. The new width is $1.2w$ and the new cost is $600(1.2w)^3 = 1.2^3 \times 600w^3 = 1.728C$. The percentage increase is $(1.728 - 1) \times 100\% = 72.8\%$.	3	
	(c) 3 cm Solve $600w^3 = 16\,200$. Then $w^3 = 27$, so $w = 3$. The width is 3 cm.	2	
6.	**(a)** 48 lux Let $E = \frac{k}{d^2}$. Substituting $d = 2$ and $E = 300$: $300 = \frac{k}{4}$, so $k = 1200$. Hence $E = \frac{1200}{d^2}$. When $d = 5$: $E = \frac{1200}{25} = 48$. The illuminance is 48 lux.	2	
	(b) 2.5 m Set $\frac{1200}{d^2} = 192$. Then $d^2 = \frac{1200}{192} = 6.25$, so $d = 2.5$ (since $d > 0$). The distance is 2.5 m.	2	
	(c) 36% The new distance is $1.25d$. The new illuminance is $\frac{k}{(1.25d)^2} = \frac{1}{1.25^2} \times \frac{k}{d^2} = \frac{1}{1.5625}E = 0.64E$, where E is the original illuminance. The percentage decrease is $(1 - 0.64) \times 100\% = 36\%$.	2	

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Question	Solution	Marks	Remarks
	(d) No; the illuminance becomes 4 times as large. At $d = 4$: $E = \frac{1200}{16} = 75$ lux. At $d = 2$: $E = \frac{1200}{4} = 300$ lux. Since $300 = 4 \times 75$, halving the distance makes the illuminance four times as large, not twice as large. The claim is incorrect.	2	
7.	**(a)** $d = 6 + 0.01v^2$ Let $d = c + kv^2$, where c is the constant reaction distance and k is a constant. Substituting the two pairs of values gives $22 = c + 1600k$ and $42 = c + 3600k$. Subtracting the first equation from the second: $20 = 2000k$, so $k = 0.01$. Then $22 = c + 16$, so $c = 6$. Hence $d = 6 + 0.01v^2$.	3	
	(b) 70 m When $v = 80$: $d = 6 + 0.01(80)^2 = 6 + 0.01(6400) = 6 + 64 = 70$. The stopping distance is 70 m.	2	
	(c) 97.0 km/h Set $6 + 0.01v^2 = 100$. Then $0.01v^2 = 94$, so $v^2 = 9400$. Since $v > 0$, taking the positive square root gives $v \approx 96.9536$, i.e. 97.0 km/h (3 s.f.).	2	
	(d) 156 m At 100 km/h the reaction distance is 6 m and the dry braking distance is $0.01(100)^2 = 100$ m. On a wet road the braking distance becomes $1.5 \times 100 = 150$ m, so $d = 6 + 150 = 156$. The stopping distance is 156 m.	2	
SECTION B (10 marks)			
8.	**(a)** 800 W Let $P = kr^2v^3$. Substituting $r = 2$, $v = 5$ and $P = 1000$: $1000 = k(2)^2(5)^3 = 500k$, so $k = 2$. Hence $P = 2r^2v^3$. When $r = 2.5$ and $v = 4$: $P = 2(2.5)^2(4)^3 = 2 \times 6.25 \times 64 = 800$. The power output is 800 W.	2	
	(b) 72.8% The new wind speed is $1.2v$, so the new power is $kr^2(1.2v)^3 = 1.2^3 \times kr^2v^3 = 1.728P$, where P is the original power output. The percentage increase is $(1.728 - 1) \times 100\% = 72.8\%$.	2	

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	(c) 216 W When $r = 2$ and $v = 3$: $P = 2(2)^2(3)^3 = 2 \times 4 \times 27 = 216$. The power output is 216 W.	2	
	(d) 7.28% Let the original blade radius and wind speed be r and v, and let the required new wind speed be v'. After the reduction the radius is $0.9r$. For the power output to be unchanged: $k(0.9r)^2(v')^3 = kr^2v^3$. Hence $0.81(v')^3 = v^3$, so $\left(\frac{v'}{v}\right)^3 = \frac{1}{0.81}$. Taking cube roots gives $\frac{v'}{v} \approx 1.07277$. The required percentage increase is $(1.07277 - 1) \times 100\% \approx 7.28\%$ (3 s.f.).	4	

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