MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: More about Equations · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$(x, y) = (3, 5)$ **or** $(-1, -3)$ Substitute $y = 2x - 1$ into $y = x^2 - 4$: $2x - 1 = x^2 - 4 \implies x^2 - 2x - 3 = 0$. Factorising, $(x - 3)(x + 1) = 0$, so $x = 3$ or $x = -1$. When $x = 3$, $y = 2(3) - 1 = 5$. When $x = -1$, $y = 2(-1) - 1 = -3$. The solutions are $(x, y) = (3, 5)$ and $(x, y) = (-1, -3)$.	3	
2.	$x = -3, -2, 2$ **or** 3 Let $u = x^2$. Then the equation becomes $u^2 - 13u + 36 = 0$, so $(u - 4)(u - 9) = 0$, giving $u = 4$ or $u = 9$. Since $u = x^2$, we have $x^2 = 4$ or $x^2 = 9$. Hence $x = \pm 2$ or $x = \pm 3$. All four values satisfy the original equation.	3	
3.	**(a)** $x = -1$ or $x = 3$ The curve cuts the x-axis at $x = -1$ and $x = 3$. Therefore the solutions are $x = -1$ or $x = 3$.	1	
	(b) $x = -2$ or $x = 4$ The line $y = 5$ meets the curve where $x^2 - 2x - 3 = 5$, that is, where $x^2 - 2x - 8 = 0$. From the graph, the two points of intersection have x-coordinates -2 and 4. Hence $x = -2$ or $x = 4$.	3	

TUTGATHER

Question	Solution	Marks	Remarks
4.	**(a)** $n(n+2) = 288$ There are n rows and $(n+2)$ chairs in each row. Since there are 288 chairs in total, $n(n+2) = 288.$	2	
	(b) 16 rows with 18 chairs in each row Expanding the equation: $n^2 + 2n - 288 = 0.$ Factorising, $(n+18)(n-16) = 0,$ so $n = -18$ or $n = 16$. Since the number of rows must be positive, $n = -18$ is rejected. Hence there are 16 rows, and the number of chairs in each row is $16 + 2 = 18$. Check: $16 \times 18 = 288$.	4	
SECTION A(2) (24 marks)			
5.	**(a)** $m < -2 - 2\sqrt{2}$ or $m > -2 + 2\sqrt{2}$ At a point of intersection, the y-coordinates are equal: $x^2 - 2x + 5 = mx + 3.$ Rearranging, $x^2 - (m+2)x + 2 = 0.$ For two distinct points of intersection, the discriminant is positive: $\Delta = [-(m+2)]^2 - 4(1)(2) = (m+2)^2 - 8 > 0.$ Thus $(m+2)^2 > 8$, so $m + 2 < -2\sqrt{2} \quad \text{or} \quad m + 2 > 2\sqrt{2}.$ Hence $m < -2 - 2\sqrt{2} \quad \text{or} \quad m > -2 + 2\sqrt{2}.$	4	

TUTGATHER

Question	Solution	Marks	Remarks
	(b) $(1, 4)$ Since $(2, 5)$ lies on L, $$5 = 2m + 3,$$ so $m = 1$. Hence L is $y = x + 3$. At an intersection, $$x^2 - 2x + 5 = x + 3,$$ so $$x^2 - 3x + 2 = 0,$$ that is, $(x - 1)(x - 2) = 0$. Thus $x = 1$ or $x = 2$. The value $x = 2$ gives the point $(2, 5)$. When $x = 1$, $y = 1 + 3 = 4$. Therefore the other point of intersection is $(1, 4)$.	3	
6.	**(a)** $x^2 - 4x - 6 = 0$ For $x \neq 1$ and $x \neq -2$, multiply both sides by $(x - 1)(x + 2)$: $$3(x + 2) + 2(x - 1) = (x - 1)(x + 2).$$ Expanding both sides, $$3x + 6 + 2x - 2 = x^2 + x - 2,$$ so $$5x + 4 = x^2 + x - 2.$$ Hence $$x^2 - 4x - 6 = 0.$$	3	
	(b) $x = 2 + \sqrt{10}$ or $x = 2 - \sqrt{10}$ Using the quadratic formula with $a = 1$, $b = -4$ and $c = -6$: $$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2} = \frac{4 \pm \sqrt{40}}{2} = 2 \pm \sqrt{10}.$$ Both values are different from 1 and −2, so both satisfy the original equation.	3	

TUTGATHER

Question	Solution	Marks	Remarks
	(c) $x = \sqrt{2+\sqrt{10}}$ or $x = -\sqrt{2+\sqrt{10}}$ Let $u = x^2$. The equation becomes $$\frac{3}{u-1} + \frac{2}{u+2} = 1,$$ which, by part (a), gives $$u^2 - 4u - 6 = 0.$$ Hence $u = 2 \pm \sqrt{10}$. Since $u = x^2 \geq 0$ and $2 - \sqrt{10} < 0$, the value $2 - \sqrt{10}$ must be rejected. Therefore $$x^2 = 2 + \sqrt{10},$$ so $$x = \pm\sqrt{2+\sqrt{10}}.$$	2	
7.	**(a)** $x \approx 4.2$ or $x \approx -0.2$ The curve cuts the x-axis at two points. Reading from the graph, their x-coordinates are approximately 4.2 and −0.2. (Exactly, $-x^2 + 4x + 1 = 0$ gives $x = 2 \pm \sqrt{5}$, so $x \approx 4.236$ or $x \approx -0.236$.)	2	
	(b) $x = 1$ or $x = 2$ From $y = -x^2 + 4x + 1$, we have $$x^2 = 4x + 1 - y.$$ Substitute this into $x^2 - 3x + 2 = 0$: $$(4x + 1 - y) - 3x + 2 = 0,$$ so $$y = x + 3.$$ Draw the line $y = x + 3$ on the graph. It meets the curve at $(1, 4)$ and $(2, 5)$, so the solutions are $x = 1$ or $x = 2$.	4	

TUTGATHER

Question	Solution	Marks	Remarks
	(c) $k < \frac{13}{4}$ The line $y = x + k$ has slope 1, so it is parallel to the line $y = x + 3$ drawn in part (b). From the graph, such a line cuts the curve twice when it lies below the tangent line of slope 1. At an intersection, $$-x^2 + 4x + 1 = x + k,$$ so $$x^2 - 3x + (k - 1) = 0.$$ For two distinct real roots, $$\Delta = (-3)^2 - 4(1)(k - 1) > 0.$$ Thus $$9 - 4k + 4 > 0,$$ so $$13 > 4k,$$ that is, $$k < \frac{13}{4}.$$ (At $k = \frac{13}{4}$ the line is tangent to the curve, giving exactly one repeated root.)	3	
SECTION B (10 marks)			
8.	**(a)** $\frac{1}{x} + \frac{1}{x+4} = \frac{5}{24}$ In one hour, pipe B fills $\frac{1}{x}$ of the pool, while pipe A fills $\frac{1}{x+4}$ of the pool. Together they fill the pool in 4.8 hours, so in one hour they fill $$\frac{1}{4.8} = \frac{5}{24}$$ of the pool. Hence $$\frac{1}{x} + \frac{1}{x+4} = \frac{5}{24}.$$ Multiplying both sides by $24x(x + 4)$: $$24(x + 4) + 24x = 5x(x + 4),$$ which simplifies to $$5x^2 - 28x - 96 = 0.$$	3	

TUTGATHER

Question	Solution	Marks	Remarks
	(b) $x = 8$ From part (a), $$5x^2 - 28x - 96 = 0.$$ Using the quadratic formula, $$x = \frac{28 \pm \sqrt{(-28)^2 - 4(5)(-96)}}{2(5)} = \frac{28 \pm \sqrt{784 + 1920}}{10} = \frac{28 \pm 52}{10}.$$ Hence $x = 8$ or $x = -2.4$. Since x represents the time taken by pipe B, it must be positive. Therefore $x = -2.4$ is rejected, leaving $x = 8$.	3	
	(c) 12 hours Pipe A takes $x + 4$ hours, so it takes $8 + 4 = 12$ hours.	1	
	(d) 6 hours In 2 hours, pipe A fills $$\frac{2}{12} = \frac{1}{6}$$ of the pool, so $\frac{5}{6}$ of the pool remains. With both pipes open, the pool fills at the rate $$\frac{1}{8} + \frac{1}{12} = \frac{3}{24} + \frac{2}{24} = \frac{5}{24}$$ per hour. The time needed to fill the remaining $\frac{5}{6}$ is $$\frac{5/6}{5/24} = \frac{5}{6} \times \frac{24}{5} = 4 \text{ hours.}$$ Hence the total time is $2 + 4 = 6$ hours.	3	

TUTGATHER