MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: More about Polynomials · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**Quotient** $3x^2 + 5x - 2$**; remainder** 2 Divide term by term. Since $3x^3 \div x = 3x^2$ and $3x^2(x-1) = 3x^3 - 3x^2$, subtracting gives $5x^2 - 7x + 4$. Next, $5x^2 \div x = 5x$ and $5x(x-1) = 5x^2 - 5x$, so subtracting gives $-2x + 4$. Finally, $-2x \div x = -2$ and $-2(x-1) = -2x + 2$, leaving a remainder of 2. Hence $3x^3 + 2x^2 - 7x + 4 = (x-1)(3x^2 + 5x - 2) + 2$.	3	
2.	**(a)** $a = 1$ By the remainder theorem, the remainder when $f(x)$ is divided by $x - 1$ is $f(1)$. Now $f(1) = 2(1)^3 + a(1)^2 - 5(1) + 6 = a + 3$. Since the remainder is 4, $a + 3 = 4$, so $a = 1$.	2	
	(b) 10 With $a = 1$, $f(x) = 2x^3 + x^2 - 5x + 6$. By the remainder theorem, the remainder on division by $x + 1$ is $f(-1) = 2(-1)^3 + (-1)^2 - 5(-1) + 6 = -2 + 1 + 5 + 6 = 10$.	1	
3.	**(a)** $a = 3$ By the factor theorem, $f(-2) = 0$. Now $f(-2) = 2(-2)^3 + a(-2)^2 - 5(-2) - 6 = -16 + 4a + 10 - 6 = 4a - 12$. Hence $4a - 12 = 0$, so $a = 3$.	2	
	(b) $f(x) = (x + 2)(2x - 3)(x + 1)$ With $a = 3$, $f(x) = 2x^3 + 3x^2 - 5x - 6$. Dividing by $x + 2$ gives $f(x) = (x + 2)(2x^2 - x - 3)$. Factorising the quadratic, $2x^2 - x - 3 = (2x - 3)(x + 1)$. Hence $f(x) = (x + 2)(2x - 3)(x + 1)$.	2	

TUTGATHER

Question	Solution	Marks	Remarks
4.	**(a)** $P(x) = x(x-2)(x+2)$; $Q(x) = (x+3)(x-2)$ $P(x) = x(x^2-4) = x(x-2)(x+2)$. Also $Q(x) = x^2 + x - 6 = (x+3)(x-2)$.	2	
	(b) $x - 2$ The only common factor of $P(x) = x(x-2)(x+2)$ and $Q(x) = (x+3)(x-2)$ is $x - 2$. Hence the H.C.F. is $x - 2$.	2	
	(c) $x(x-2)(x+2)(x+3)$ The L.C.M. must contain each factor that appears, to its highest power: x, $x - 2$, $x + 2$ and $x + 3$. Hence the L.C.M. is $x(x-2)(x+2)(x+3)$.	2	
SECTION A(2) (24 marks)			
5.	**(a)** $(42x - 21)$ cm^2 The area of the frame is $(3x+2)^2 - (3x-5)^2$. Taking $a = 3x + 2$ and $b = 3x - 5$ in $a^2 - b^2 = (a-b)(a+b)$, the area is $[(3x+2) - (3x-5)][(3x+2) + (3x-5)] = (7)(6x-3) = 42x - 21$. Hence the area is $(42x - 21)$ cm^2.	2	
	(b) $x = 6$ Set $42x - 21 = 231$. Then $42x = 252$, so $x = 6$.	2	
	(c) 4221 Observe that $305 = 3(101) + 2$ and $298 = 3(101) - 5$. Taking $x = 101$ in the identity in (a), $305^2 - 298^2 = 42(101) - 21 = 4242 - 21 = 4221$.	3	
6.	**(a)** Quotient $3x^2 + 5x - 2$; remainder 0 $3x^3 \div x = 3x^2$ and $3x^2(x-1) = 3x^3 - 3x^2$, leaving $5x^2 - 7x + 2$. Next, $5x^2 \div x = 5x$ and $5x(x-1) = 5x^2 - 5x$, leaving $-2x + 2$. Finally, $-2x \div x = -2$ and $-2(x-1) = -2x + 2$, leaving remainder 0. Hence $3x^3 + 2x^2 - 7x + 2 = (x-1)(3x^2 + 5x - 2)$.	3	
	(b) $f(x) = (x-1)(3x-1)(x+2)$ Factorising the quotient, $3x^2 + 5x - 2 = (3x-1)(x+2)$. Hence $f(x) = (x-1)(3x-1)(x+2)$.	2	

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Question	Solution	Marks	Remarks
	(c) $x = 1$, $x = \frac{1}{3}$ or $x = -2$ From $(x-1)(3x-1)(x+2) = 0$, we get $x = 1$, $x = \frac{1}{3}$ or $x = -2$.	1	
	(d) 0 From part (b), $3x - 1$ is a factor of $f(x)$, so the remainder is 0. Equivalently, by the remainder theorem, $f(\frac{1}{3}) = 0$.	2	
7.	**(a)** $k = -1$ By the remainder theorem, the two remainders are $f(1) = 1 + k - 4 - 6 = k - 9$ and $f(-2) = -8 + 4k + 8 - 6 = 4k - 6$. Since the remainders are equal, $k - 9 = 4k - 6$. Hence $-3 = 3k$, so $k = -1$.	4	
	(b) -10 With $k = -1$, the common remainder is $f(1) = 1 - 1 - 4 - 6 = -10$. As a check, $f(-2) = 4(-1) - 6 = -10$.	2	
	(c) $f(3) = 0$, so $x - 3$ is a factor With $k = -1$, $f(x) = x^3 - x^2 - 4x - 6$. Then $f(3) = 3^3 - 3^2 - 4(3) - 6 = 27 - 9 - 12 - 6 = 0$. By the factor theorem, $x - 3$ is a factor of $f(x)$.	3	
SECTION B (10 marks)			
8.	**(a)** $a = -1$, $b = -1$, $c = 1$ Since $P(1) = 0$, $1 + a + b + c = 0$. Since $P(-1) = 0$, $-1 + a - b + c = 0$. Subtracting the second equation from the first gives $2 + 2b = 0$, so $b = -1$. Adding the two equations gives $2a + 2c = 0$, so $c = -a$. Finally, $P(2) = 3$ gives $8 + 4a + 2b + c = 3$, so $8 + 4a - 2 + c = 3$, i.e. $4a + c = -3$. Substituting $c = -a$ gives $3a = -3$, so $a = -1$ and $c = 1$. Hence $a = -1$, $b = -1$ and $c = 1$.	4	
	(b) $P(x) = (x-1)^2(x+1)$ With $a = -1$, $b = -1$ and $c = 1$, $P(x) = x^3 - x^2 - x + 1$. Since $P(1) = 0$, $x - 1$ is a factor. Dividing, $P(x) = (x-1)(x^2 - 1) = (x-1)(x-1)(x+1) = (x-1)^2(x+1)$. Hence $(x-1)^2$ is a factor of $P(x)$.	3	

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Question	Solution	Marks	Remarks
	(c) $x = 1$ (repeated) or $x = -1$ From $(x-1)^2(x+1) = 0$, we get $x = 1$ (repeated) or $x = -1$.	1	
	(d) The graph crosses the x-axis at $x = -1$ and touches it at $x = 1$. Since $P(x) = (x-1)^2(x+1)$, near $x = 1$ the factor $(x-1)^2$ is non-negative and does not change sign, so $P(x)$ has the same sign on both sides of 1 and the graph touches the x-axis there. Near $x = -1$ the factor $x + 1$ changes sign while $(x-1)^2 > 0$, so $P(x)$ changes sign and the graph crosses the x-axis there.	2	

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