MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: Exponential and Logarithmic Functions · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	$\frac{3x^5}{y^4}$ Using the laws of indices, $(27x^6y^{-3})^{2/3} = 27^{2/3}x^4y^{-2} = 9x^4y^{-2}$ and $(9x^{-2}y^4)^{1/2} = 9^{1/2}x^{-1}y^2 = 3x^{-1}y^2$. Hence the expression equals $\frac{9x^4y^{-2}}{3x^{-1}y^2} = 3x^5y^{-4} = \frac{3x^5}{y^4}$.	3	
2.	$k = 5$, $a = 3$; **when** $x = -1$, $y = \frac{5}{3}$ Substituting $(0, 5)$ into $y = ka^x$ gives $5 = ka^0 = k$, so $k = 5$. Substituting $(2, 45)$ gives $45 = 5a^2$, so $a^2 = 9$. Since $a > 0$, $a = 3$. When $x = -1$, $y = 5(3^{-1}) = \frac{5}{3}$.	3	
3.	$\frac{5}{12}$ Write each logarithm as a power. Since $8^{2/3} = 4$, $\log_8 4 = \frac{2}{3}$. Since $9^{-3/2} = \frac{1}{27}$, $\log_9 \frac{1}{27} = -\frac{3}{2}$. Since $16^{5/4} = 32$, $\log_{16} 32 = \frac{5}{4}$. Hence the sum is $\frac{2}{3} - \frac{3}{2} + \frac{5}{4} = \frac{8 - 18 + 15}{12} = \frac{5}{12}$.	4	
4.	**(a)** $3x + 2y$ $\log 72 = \log(2^3 \cdot 3^2) = 3\log 2 + 2\log 3 = 3x + 2y$.	3	
	(b) $\frac{3x + y}{x + y}$ By the change of base formula, $\log_6 24 = \frac{\log 24}{\log 6} = \frac{\log(2^3 \cdot 3)}{\log(2 \cdot 3)} = \frac{3\log 2 + \log 3}{\log 2 + \log 3} = \frac{3x + y}{x + y}$.	3	
SECTION A(2) (24 marks)			
5.	**(a)** $x = 1$ or $x = 3$ Let $u = 2^x$. Since $2^{2x} = u^2$ and $2^{x+1} = 2u$, the equation becomes $u^2 - 10u + 16 = 0$, i.e. $(u - 2)(u - 8) = 0$. Thus $u = 2$ or $u = 8$, giving $2^x = 2$ or $2^x = 8$. Hence $x = 1$ or $x = 3$.	4	

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Question	Solution	Marks	Remarks
	(b) $x = 5$ For the logarithms to be defined, $x + 3 > 0$ and $x - 3 > 0$, so $x > 3$. Combining the logarithms gives $\log_4[(x + 3)(x - 3)] = 2$, so $(x + 3)(x - 3) = 4^2 = 16$. Hence $x^2 - 9 = 16$, i.e. $x^2 = 25$, so $x = 5$ or $x = -5$. Since $x > 3$, reject $x = -5$. Therefore $x = 5$.	3	
6.	**(a)** HK$57 732 The monthly interest rate is $\frac{3.6\%}{12} = 0.3\% = 0.003$. After 48 months, the amount is $50000(1.003)^{48} \approx 57731.76$. Hence the amount is HK$57 732 (to the nearest dollar).	2	
	(b) 61 months We need $50000(1.003)^n > 60000$, i.e. $(1.003)^n > 1.2$. Taking logarithms gives $n\log1.003 > \log1.2$, so $n > \frac{\log1.2}{\log1.003} \approx 60.86$. Hence the least integer n is 61. Check: $50000(1.003)^{60} \approx 59844.74 < 60000$ and $50000(1.003)^{61} \approx 60024.27 > 60000$.	3	
	(c) 3.66% per annum; interest is credited monthly, so interest earned during the year itself earns further interest. After one year, an initial amount P grows to $P(1.003)^{12}$, so the effective annual rate is $(1.003)^{12} - 1 \approx 0.03659998$, i.e. 3.66% per annum (correct to 2 decimal places). It is greater than the nominal rate 3.6% because interest is added to the account every month and therefore earns further interest during the remaining months of the year.	3	
7.	**(a)** $x^{1/2} + x^{-1/2} = 3$ Expanding, $(x^{1/2} + x^{-1/2})^2 = x + 2x^{1/2}x^{-1/2} + x^{-1} = x + 2 + \frac{1}{x} = 7 + 2 = 9$. Since $x > 0$, $x^{1/2} + x^{-1/2} > 0$, so $x^{1/2} + x^{-1/2} = \sqrt{9} = 3$.	3	
	(b) 18 Expanding, $(x^{1/2} + x^{-1/2})(x + x^{-1}) = x^{3/2} + x^{-1/2} + x^{1/2} + x^{-3/2} = (x^{3/2} + x^{-3/2}) + (x^{1/2} + x^{-1/2})$. Hence $3 \times 7 = (x^{3/2} + x^{-3/2}) + 3$, so $x^{3/2} + x^{-3/2} = 21 - 3 = 18$.	3	
	(c) 123 Similarly, $(x^{3/2} + x^{-3/2})(x + x^{-1}) = x^{5/2} + x^{-5/2} + x^{1/2} + x^{-1/2} = (x^{5/2} + x^{-5/2}) + (x^{1/2} + x^{-1/2})$. Thus $18 \times 7 = (x^{5/2} + x^{-5/2}) + 3$, so $x^{5/2} + x^{-5/2} = 126 - 3 = 123$.	3	

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Question	Solution	Marks	Remarks
SECTION B (10 marks)			
8.	**(a)** $f(x) = 2^x - 2^{-x}$ $f(x) = \frac{4^x}{2^x} - \frac{1}{2^x} = 2^{2x-x} - 2^{-x} = 2^x - 2^{-x}$.	2	
	(b) $f(-x) = -f(x)$; the graph has rotational symmetry of order 2 about the origin. $f(-x) = 2^{-x} - 2^{-(-x)} = 2^{-x} - 2^x = -(2^x - 2^{-x}) = -f(x)$. Hence the graph of $y = f(x)$ is unchanged when rotated through 180° about the origin; equivalently, it has point symmetry about the origin.	2	
	(c) $x = 1$ Let $u = 2^x$, where $u > 0$. Then $u - \frac{1}{u} = \frac{3}{2}$, so $2u^2 - 3u - 2 = 0$. Factorising gives $(2u + 1)(u - 2) = 0$, so $u = -\frac{1}{2}$ or $u = 2$. Since $u > 0$, $u = 2$, giving $2^x = 2$ and hence $x = 1$.	3	
	(d) For each real k, the equation has exactly one positive root u in $u^2 - ku - 1 = 0$, and $2^x = u$ has exactly one real solution. Put $u = 2^x$, where $u > 0$. The equation becomes $u - \frac{1}{u} = k$, i.e. $u^2 - ku - 1 = 0$. Its discriminant is $k^2 + 4 > 0$, so it has two distinct real roots, and their product is $-1 < 0$; hence exactly one root is positive. For this positive root u, the equation $2^x = u$ has exactly one real solution $x = \log_2 u$. Therefore $f(x) = k$ has exactly one real solution for every real k.	3	

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