MARKING SCHEME

MATHEMATICS Compulsory Part · Paper 1

Topic practice: Functions and Graphs · 2027

This marking scheme has been prepared for markers' reference. It is not a set of model answers. Award method marks where the working is sound. Accept equivalent methods unless the question specifies otherwise.

Question	Solution	Marks	Remarks
SECTION A(1) (16 marks)			
1.	**(a)** $f(-2) = 3$ $f(-2) = \sqrt{5-2(-2)} = \sqrt{5+4} = \sqrt{9} = 3$.	1	
	(b) $x \le \frac{5}{2}$, i.e. $(-\infty, \frac{5}{2}]$ For $f(x)$ to be a real number, the expression under the square root must be non-negative: $5 - 2x \ge 0$. Hence $2x \le 5$, so $x \le \frac{5}{2}$. The domain is $(-\infty, \frac{5}{2}]$.	2	
2.	**(a)** $k = -2$ Substitute $x = 3$ and $y = -18$: $-18 = k(3)^2 = 9k$, so $k = -2$.	1	
	(b) The graph opens downwards; vertex $(0, 0)$. Since $k = -2 < 0$, the graph of $y = -2x^2$ opens downwards. Its highest point is the origin, so the vertex is $(0, 0)$.	2	
3.	$b = -4$, $c = 5$ From $f(1) = 3$: $2 + b + c = 3$, so $b + c = 1$ (1). From $f(-2) = 21$: $8 - 2b + c = 21$, so $-2b + c = 13$ (2). Subtracting (1) from (2): $-3b = 12$, hence $b = -4$. Then $c = 1 - (-4) = 5$. Check: $f(x) = 2x^2 - 4x + 5$, $f(1) = 3$ and $f(-2) = 8 + 8 + 5 = 21$.	4	
4.	**(a)** The y-intercept is 120. It is the call-out fee, i.e. the charge before any distance is travelled. The line meets the y-axis at $(0, 120)$, so the y-intercept is 120. It is the total charge when $x = 0$, i.e. the fixed call-out fee of \$120.	2	

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Question	Solution	Marks	Remarks
	(b) \$20 per km The line passes through $(0, 120)$ and $(10, 320)$. The charge per kilometre is the slope: $\frac{320-120}{10-0} = \frac{200}{10} = 20$. Hence the charge is \$20 per km.	2	
	(c) \$420 Using the call-out fee and the rate: total charge $= 120 + 20(15) = 120 + 300 = 420$. Hence the total charge is \$420.	2	
SECTION A(2) (24 marks)			
5.	**(a)** $f(4) = \frac{11}{3}$, $f(-1) = -\frac{1}{2}$ $f(4) = \frac{2(4)+3}{4-1} = \frac{11}{3}$. Also $f(-1) = \frac{2(-1)+3}{-1-1} = \frac{1}{-2} = -\frac{1}{2}$.	2	
	(b) $x = \frac{8}{3}$ $\frac{2x+3}{x-1} = 5$, so $2x + 3 = 5(x - 1) = 5x - 5$. Then $3 + 5 = 5x - 2x$, i.e. $8 = 3x$, so $x = \frac{8}{3}$. This value is not 1, so it is accepted.	2	
	(c) The claim is false; the range is $\{y \in R: y \neq 2\}$. Suppose $f(x) = 2$. Then $\frac{2x+3}{x-1} = 2$, so $2x + 3 = 2x - 2$, which gives $3 = -2$, a contradiction. Hence $f(x)$ can never equal 2. Conversely, for any real $y \neq 2$, solving $y = \frac{2x+3}{x-1}$ gives $x(y - 2) = y + 3$, so $x = \frac{y+3}{y-2}$, which is a real number not equal to 1. Thus every real value except 2 is attained, and the range is $\{y \in R: y \neq 2\}$.	3	
6.	**(a)** Vertex $(2, -9)$; axis of symmetry $x = 2$; x-intercepts $(-1, 0)$ and $(5, 0)$ The graph is $y = (x - 2)^2 - 9$, so the vertex is $(2, -9)$ and the axis of symmetry is $x = 2$. For the x-intercepts, set $y = 0$: $(x - 2)^2 = 9$, so $x - 2 = \pm 3$. Hence $x = 5$ or $x = -1$, giving $(-1, 0)$ and $(5, 0)$.	3	
	(b) $g(x) = -(x - 1)^2 + 13$ Reflecting in the x-axis gives $y = -f(x)$. Translating 1 unit to the left replaces x by $x + 1$, and translating 4 units upwards adds 4: $g(x) = -f(x + 1) + 4$. Now $f(x + 1) = (x + 1 - 2)^2 - 9 = (x - 1)^2 - 9$, so $g(x) = -[(x - 1)^2 - 9] + 4 = -(x - 1)^2 + 13$.	3	
	(c) $x = 1 \pm \sqrt{13}$ Set $g(x) = 0$: $-(x - 1)^2 + 13 = 0$, so $(x - 1)^2 = 13$. Hence $x - 1 = \pm\sqrt{13}$ and $x = 1 \pm \sqrt{13}$.	2	

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Question	Solution	Marks	Remarks
7.	**(a)** $P(x) = -2(x-8)^2 + 72$; maximum profit \$72 when $x = 8$ $P(x) = -2(x^2 - 16x) - 56 = -2[(x-8)^2 - 64] - 56 = -2(x-8)^2 + 128 - 56 = -2(x-8)^2 + 72$. Since $-2(x-8)^2 \le 0$, we have $P(x) \le 72$, with equality when $x = 8$. The maximum profit is \$72 at a price of \$8.	3	
	(b) $x = 2$ or $x = 14$ Set $P(x) = 0$: $-2(x-8)^2 + 72 = 0$, so $(x-8)^2 = 36$. Hence $x - 8 = \pm 6$, giving $x = 14$ or $x = 2$. The break-even prices are \$2 and \$14.	3	
	(c) $4 \le x \le 12$ Require $P(x) \ge 40$: $-2(x-8)^2 + 72 \ge 40$, so $-2(x-8)^2 \ge -32$, i.e. $(x-8)^2 \le 16$. Hence $-4 \le x - 8 \le 4$, giving $4 \le x \le 12$. The profit is at least \$40 when the price is between \$4 and \$12 inclusive.	3	
SECTION B (10 marks)			
8.	**(a)** $f(x) = x^2 - 2x - 3$ Since the x-intercepts are -1 and 3, write $f(x) = a(x+1)(x-3)$. The y-intercept is -3, so $f(0) = a(1)(-3) = -3a = -3$, giving $a = 1$. Hence $f(x) = (x+1)(x-3) = x^2 - 2x - 3$.	3	
	(b) $P(4, 5)$ At an intersection, $f(x) = 2x - 3$: $x^2 - 2x - 3 = 2x - 3$, so $x^2 - 4x = 0$, i.e. $x(x-4) = 0$. Thus $x = 0$ or $x = 4$. The point with $x = 0$ is $(0, -3)$, so P has x-coordinate 4. Its y-coordinate is $2(4) - 3 = 5$, so $P(4, 5)$.	2	
	(c) $0 < x < 4$ The graph of $y = f(x)$ lies below the line L between the two intersection points, whose x-coordinates are 0 and 4. Hence $f(x) < 2x - 3$ when $0 < x < 4$.	2	
	(d) $k = 4$ The graphs meet where $f(x) + k = 2x - 3$: $x^2 - 2x - 3 + k = 2x - 3$, so $x^2 - 4x + k = 0$. Completing the square gives $(x-2)^2 + k - 4 = 0$. There is exactly one solution when $k - 4 = 0$, i.e. $k = 4$. (The point of contact is $(2, 1)$.)	3	

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